# Massless charged particles, naked singularity, and GUP in Reissner-Nordström-de Sitter-like spacetime

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Motivated by the endeavors of Li Xiang and You-Gen Shen on naked singularities, we investigate the validity of the cosmic censorship conjecture in the context of the generalized uncertainty principle. In particular, upon considering both linear and quadratic terms of momentum in the uncertainty principle, we first compute the entropy of a massless charged black hole in de Sitter spacetime at a given modified temperature. Then, we compute the corresponding modified cosmological radius and express the black hole electric charge in terms of this modified cosmological radius and, thus, in terms of the generalized uncertainty principle parameter. Finally, we examine whether such a system will end up being a naked singularity or might be protected by the cosmic censorship conjecture and how that might be related to the possible existence of massless charged particles.

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## I. INTRODUCTION

General Relativity (GR) is so successful, not only because of its accurate predictions but also its capability of predicting its shortcomings. One of the "apparent" shortcomings of GR is that it predicts the existence of *singularities*, spacetime geometrical points where curvature becomes infinite and laws of physics are no longer working. Einstein abhorred their occurrence [1]. He believed they are just made-up ramifications of spacetime symmetries and morphisms. However, it turned out that they are inescapable through Penrose-Hawking singularity theorems [2–5]. It is worth noting that the *classical* Einstein-Maxwell equations were introduced as a deterministic unified theory. However, it was shown that the Einstein-Maxwell equations can avoid singularities through, interestingly, violating the energy density condition of positivity of Penrose-Hawking singularity theorems [6,7]. Nevertheless, this violation has undesired consequences [8,9] upon considering the positive energy theorem [10–12]. So, the moral is that GR, alone or combined with other classical theories, fails to deal with singularities. However, this is not truly considered a shortcoming as long as the singularity is *causally* separated—and hence impossible to observe—from the rest of the Universe through an event horizon, a "doorstep" at which timelike and spacelike coordinates interchange their rules. That makes static uncharged black hole singularities spacelike points; meanwhile, those of rotating and/or charged black holes may disrupt the causal structure of spacetime. Interestingly, since Penrose-Hawking singularity theorems do not say much about geometrical locations of singularities, then the existence of singularity is not necessarily accompanied within black hole structure [13]; i.e., they might not be in need of an event horizon. These singularities are called naked singularities [14–17]. Since a naked singularity lacks that doorstep, timelike and spacelike coordinates keep their geometrical rules unchanged around that point. This would lead to a major breakdown of foundations of spacetime geometry. Consequently, the "realistic" perspectives on classical theories of physics would be demolished. Being a Platonist[1][18], Penrose introduced the cosmic censorship conjecture [3]: "Nature abhors a naked singularity".

Since the introduction of cosmic censorship conjecture, many endeavors have attempted to argue in favor of—mainly by focusing on the Cauchy horizon [19–27]—or against the weak and strong versions of cosmic censorship

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[1]Hawking meant "Platonic realist." Like Einstein, Penrose was concerned about maintaining the *deterministic* predictability of GR as a local theory. This philosophical stance is based on the fact that GR is exclusively a geometric theory of Lorentzian manifolds.

 

conjecture [28]—mainly by focusing on gravitational dust collapse processes [29–38]. None came with conclusive definitive proof of whether naked singularities could or could not physically exist and a question is raised if it is necessary to change the methodology which was used to deal with the cosmic censorship conjecture [39]. Another line of research that deals with this conjecture is to see its *topological* effects together with null energy condition [40–53]. Also, there is another debate about the validity of either cosmic censorship conjecture or naked singularities in modified gravity theories upon considering the null energy condition [54–62]. Recent *analytical* proof [63] is in favor of the cosmic censorship conjecture, while others [64–66] counterargued the cosmic censorship conjecture, leaving the question unanswered.

The biggest challenge in contemporary physics is to unify GR and quantum mechanics (QM) in a concrete theory of quantum gravity. A little bit auspicious way, of many, to seize the "holy grail" is to construct a quantum field theory in curved spacetime [67,68], in which the curved spacetimes are black holes. This approach succeeded in introducing Hawking radiation and black hole entropy [69,70] and the black hole information riddle [71]. Before that, Wald's classical gedanken experiment failed to destroy the event horizon by overcharging the corresponding Kerr-Newman black hole [72]. If the experiment had succeeded, then—upon considering the quantum effects—the resulting naked singularity would have absorbed all black hole entropy. This would contradict the holographic principle [73,74], as a naked singularity of Planck length size $\ell_p$ can carry only a few bits of information. More recent classical gedanken experiments also support the cosmic censorship conjecture upon considering a different Reissner-Nordström black hole [75] or upon considering the same Kerr-Newman black hole with overcharge and overspin together [76]. So, within the conservation of the information paradigm, "stripping" singularity would provoke vehement information loss [77]. It is worth noting that despite the fact that the metric at the singularity is no longer *regular*, i.e., it is degenerate, information may be retrieved out of the singularity even if the used technique does not develop the initial formulation [78].

So until the "advent" of a mature, consistent, and complete theory of quantum gravity takes place, the question of the cosmic censorship conjecture remains open. But, generally, it is believed that the GR and QM "marriage" would happen upon some compromises. It could be necessary that QM laws need some tweaks, e.g., modifying the Heisenberg uncertainty principle (HUP), to be compatible with a fundamental characteristic of string theory, that is, energy corresponds with a UV/IR increment in its length. That leads us to introduce a generalized uncertainty principle (GUP) [79–88] as another attempt to reconcile GR and QM.

In the remainder of this work, we summarize the endeavors of Li and Shen [89] as well as of Xiang and Shen [90] to examine the effect of the quadratic GUP on the cosmic censorship conjecture. Then, we introduce both a linear and quadratic GUP to compute the GUP-modified entropy for a static spherically symmetric black hole in de Sitter spacetime. For the specific case of the massless charged Reissner-Nordström-de Sitter (RNdS) spacetime, we find the location of the cosmological horizon and also show that there are no more horizons, so the curvature singularity is a naked singularity. Thus, one can assume that the Hawking radiation consists of massless charged particles. Therefore, we calculate the total energy density of those massless charged particles. Finally, the results with some concluding comments are presented. Here, natural units will be used, i.e., $\hbar = c = k_B = 1$.

## II. QUADRATIC GUP EFFECTS ON RNdS-LIKE SPACETIME

In this section, we summarize the analysis of Refs. [89,90], starting by considering the quadratic GUP to be given by

$$\Delta x \Delta p \geq 1 + \lambda \Delta p^2, \tag{1}$$

which gives an uncertainty in momentum,

$$\Delta p \sim \frac{\Delta x - \sqrt{\Delta x^2 - 4\lambda}}{2\lambda}, \tag{2}$$

where $\lambda$ is the dimensionful GUP parameter that is proportional to the squared Planck length $\ell_p^2$ with $\ell_p \sim \sqrt{G}$ and $G$ is Newton's gravitational constant. According to Planckian thermodynamics, $p \sim E \sim T$. Therefore, in the presence of a quantum black hole of event horizon radius $r_h$, Eq. (2) can be read as

$$T \sim \frac{r_h - \sqrt{r_h^2 - 4\lambda}}{2\lambda}. \tag{3}$$

Similarly, for time and energy conjugates, we have

$$\Delta E \sim \frac{\Delta t - \sqrt{\Delta t^2 - 4\lambda}}{2\lambda}, \tag{4}$$

where the signs in front of the radicals in Eqs. (2), (3), and (4) were selected such that when we take the $\lambda \to 0$ limit we then obtain the conventional HUP,

$$\Delta p \sim \frac{1}{\Delta x} \quad \text{and} \quad \Delta E \sim \frac{1}{\Delta t}. \tag{5}$$

At this point, it is worth making the following comment. The GUP corrections can be assigned to the Planck constant,[2] i.e., $\hbar$, and thus we can define an *effective* Planck constant as

$$\hbar' \sim \hbar(1+\lambda\Delta E^2). \tag{6}$$

In this case, the Hawking temperature, i.e., $T_H = \frac{\hbar}{\beta}$, is modified and becomes

$$T'_H \sim \frac{\hbar(1+\lambda\Delta E^2)}{\beta}, \tag{7}$$

where $\beta$ is called the *reciprocal temperature* and is given as $\beta = 2\pi\kappa^{-1}$ with $\kappa$ to be the surface gravity of the black hole horizon.

Since the surface gravity is inversely proportional to the black hole radius, i.e., $\kappa \sim r_h^{-1}$, the reciprocal temperature will be proportional to the black hole radius, i.e., $\beta \sim r_h$, and thus the temperature given in Eq. (3) becomes

$$T'_H = \frac{\beta - \sqrt{\beta^2 - 4\lambda}}{2\lambda} \tag{8}$$

$$= \frac{2}{\beta + \sqrt{\beta^2 - 4\lambda}}. \tag{9}$$

The first law of black hole mechanics [91] for a Schwarzschild black hole in de Sitter space [89] reads

$$dM = -\frac{\kappa_c}{8\pi} dA_c - \frac{V}{8\pi} d\Lambda, \tag{10}$$

where the subscript c denotes the *cosmological* horizon, $\Lambda$ is the cosmological constant, and $V = \frac{4\pi}{3} r_c^3$ is the volume of the de Sitter universe. Now, because of the problem of negative temperatures [see the factor of the differential $dA_c$ in Eq. (10)], we redefine $M$ to be the mass of everything inside the cosmological horizon, i.e., $r_c$, including the black hole mass. This defines the total energy of such a system to be $E_0 = M + E_{\mathrm{vac}}$, and we demand this total energy to be conserved. Therefore, the first law of thermodynamics for the cosmological horizon becomes

$$dE_{\mathrm{vac}} = -dM = \frac{\kappa_c}{8\pi} dA_c + \frac{V}{8\pi} d\Lambda. \tag{11}$$

When we also demand the entropy of this system to be maximum, then the modified temperature of the black hole, i.e., $T'_H$, becomes equal to the temperature of the cosmological horizon, i.e., $T_c$, and the entropy of cosmological horizon becomes

$$S_c = \int \frac{dE_{\mathrm{vac}}}{T_c} = \frac{A_c}{4} - \int \frac{\lambda}{4\beta^2} dA_c, \tag{12}$$

where $A_c$ is cosmological area. At this point, we should point out that if we take the limit $\lambda \to 0$ we obtain the Bekenstein-Hawking entropy of the cosmological horizon in the context of HUP.

Let us now assume we have a massless charged RNdS-like spacetime with metric

$$ds^2 = -f(r,Q,\Lambda)dt^2 + \frac{dr^2}{f(r,Q,\Lambda)} + r^2 d\Omega, \tag{13}$$

with the components of the metric being

$$f(r,Q,\Lambda) = 1 + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3} \tag{14}$$

and the cosmological horizon, i.e., $r_c$, being defined as $f(r_c, Q, \Lambda) = 0$; thus, the electric charge reads

$$Q^2 = \frac{\Lambda r_c^4}{3} - r_c^2. \tag{15}$$

The corresponding surface gravity will be

$$\kappa_c = \left(\frac{2\Lambda r_c}{3} - \frac{1}{r_c}\right). \tag{16}$$

Employing Eq. (16) in order to compute the last term of Eq. (12), we get

$$\Delta S = -\frac{\lambda}{16\pi^2}\int \kappa^2 dA_c = -\frac{\lambda}{2\pi}\left(\ln r_c - \frac{2\Lambda}{3} r_c^2 + \frac{\Lambda^2}{9} r_c^4\right). \tag{17}$$

Now, we can consider the most probable $\Lambda$ associated with the maximum entropy by setting $\partial(\Delta S)/\partial\Lambda = 0$. This will give $r_c = \sqrt{3/\Lambda}$, which means that the corresponding cosmological radius is equal to the radius of de Sitter spacetime that is empty of any mass and charge. Therefore, in the context of the quadratic GUP, the second law of black hole mechanics forbids the existence of massless charged particles.

In Ref. [90], it was shown that, in systems such as the one under study here, the cosmic censorship conjecture is guaranteed by the quadratic GUP upon considering the energy-time uncertainty

$$\Delta t \geq \frac{1}{\Delta E} + \lambda \Delta E \tag{18}$$

[2]Here, only for this comment, we have reinstated the units of $\hbar$.

that imposes a bound on the rate of energy loss as

$$\frac{\Delta E}{\Delta t} \sim \frac{dE}{dt} < \frac{1}{\lambda}, \tag{19}$$

which in turn leads to

$$E \leq \frac{L}{\lambda} \sim \frac{1}{\sqrt{\lambda}}, \tag{20}$$

where $L$ is the characteristic length of the system under study. The reason why Eq. (20) keeps the cosmic censorship conjecture safe is that for a "covered" black hole singularity to be a naked singularity it is necessary for the black hole to absorb the total mass of the system, which means $E \gg m_p \sim \frac{1}{\sqrt{\lambda}}$. It is evident that this contradicts our result given by Eq. (20) in the context of quadratic GUP.

## III. LINEAR AND QUADRATIC GUP EFFECTS ON RNdS-LIKE SPACETIME

In this section, following the previous analysis, we will reexamine everything we have just summarized in Ref. [90] in the light of the linear and quadratic GUP. The linear and quadratic GUP, which is also compatible with doubly special relativity, is given as [88]

$$\Delta x \Delta p \geq 1 - \alpha \Delta p + 4\alpha^2 \Delta p^2, \tag{21}$$

where if the $\alpha$ term is neglected compared with the $\alpha^2$ term, Eq. (21) becomes Eq. (1). The corresponding GUP-modified uncertainty in momentum is of the form

$$\Delta p \sim \frac{(\Delta x + \alpha) - \sqrt{(\Delta x + \alpha)^2 - 16\alpha^2}}{8\alpha^2}, \tag{22}$$

and the GUP-modified uncertainty in energy will now read

$$\Delta E \sim \frac{(\Delta t + \alpha) - \sqrt{(\Delta t + \alpha)^2 - 16\alpha^2}}{8\alpha^2}, \tag{23}$$

with the corresponding GUP-modified temperature of the form

$$T \sim \frac{(r_h + \alpha) - \sqrt{(r_h + \alpha)^2 - 16\alpha^2}}{8\alpha^2}. \tag{24}$$

As in the previous section, one can make the comment that all linear and quadratic GUP corrections can be assigned to an effective Planck constant, which is now modified as

$$\hbar' \sim \hbar(1 - \alpha \Delta E + 4\alpha^2 \Delta E^2). \tag{25}$$

Therefore, the GUP-modified Hawking temperature becomes

$$\begin{aligned} T'_H &= \frac{(\beta + \alpha) - \sqrt{(\beta + \alpha)^2 - 16\alpha^2}}{8\alpha^2} \\ &= \frac{2}{(\beta + \alpha) + \sqrt{(\beta + \alpha)^2 - 16\alpha^2}}, \end{aligned} \tag{26}$$

which, as expected, is exactly the same as the one given in Eq. (24) since, as already mentioned, $\beta \sim r_h$.

Now, following the analysis of the previous section, inside the cosmological horizon the modified Hawking temperature, i.e., $T'_H$, becomes equal to the temperature of the cosmological horizon, i.e., $T_c$, the corresponding entropy of the cosmological horizon becomes

$$\begin{aligned} S_c &= \int \frac{dE_{\text{vac}}}{T_c} \\ &= \int \frac{(\beta + \alpha) + \sqrt{(\beta + \alpha)^2 - 16\alpha^2)}}{2} \times \frac{\kappa_c dA_c}{8\pi}. \end{aligned} \tag{27}$$

Then, we expand $S_c$ up to $\mathcal{O}(\alpha^3)$ to get

$$S_c = \frac{A_c}{4} - \int \left(\frac{\alpha^2}{\beta^2} - \frac{\alpha}{4\beta}\right) dA_c. \tag{28}$$

At this point, it should be stressed that this extra $\frac{\alpha}{2\beta}$ term will dramatically change the previous calculations for the cosmological radius and consequently everything after.

Let us now employ the metric of a massless charged RNdS-like black hole. First, upon combining Eqs. (16) and (28), the GUP-corrected entropy $\Delta S$ ends up being

$$\Delta S = -\int \left(\frac{\alpha^2}{\beta^2} - \frac{\alpha}{4\beta}\right) dA_c \tag{29}$$

$$\begin{aligned} &= \int \left[ -\frac{\alpha^2}{4\pi^2} \left( \frac{4\Lambda^2}{9} r_c^2 - \frac{4\Lambda}{3} + \frac{1}{r_c^2} \right) \right. \\ &\quad \left. + \frac{\alpha}{8\pi} \left( \frac{2\Lambda}{3} r_c - \frac{1}{r_c} \right) \right] dA_c \end{aligned} \tag{30}$$

$$\begin{aligned} &= -\frac{2\alpha^2}{\pi} \left( \frac{\Lambda^2}{9} r_c^4 - \frac{2\Lambda}{3} r_c^2 + \ln r_c \right) \\ &\quad + \alpha \left( \frac{2\Lambda}{9} r_c^3 - r_c \right). \end{aligned} \tag{31}$$

Then, maximizing the entropy with respect to $\Lambda$ in the light of the extra $\alpha$ term in $\Delta S$, the corresponding cosmological radius is determined by both $\alpha$ and $\Lambda$ together according to the equation

$$r_c^2 - \frac{\pi}{2\alpha\Lambda} r_c - \frac{3}{\Lambda} = 0, \tag{32}$$

which gives the root

$$r_c = \left(\frac{\pi}{4\alpha\Lambda}\right) + \sqrt{\left(\frac{\pi}{4\alpha\Lambda}\right)^2 + \frac{3}{\Lambda}}. \tag{33}$$

It is noteworthy that the other root in order to be positive demands $\Lambda < 0$, which of course contradicts the fact that the spacetime is de Sitter.

For the sake of comparison with the result obtained in the previous section, namely, $r_c = \sqrt{3/\Lambda}$, we expand the root given in Eq. (33) up to $\mathcal{O}(\alpha^3)$ to obtain

$$r_c \sim \left(\frac{\pi}{4\alpha\Lambda}\right) + \sqrt{\frac{3}{\Lambda}}\left[1 + \frac{1}{2}\left(\frac{\pi}{4\alpha\Lambda}\right)^2 \frac{\Lambda}{3}\right]. \tag{34}$$

At this point, a couple of comments are in order. First, when we are in strong gravity regimes, e.g., near black hole horizons, which can be viewed as $\alpha \to \infty$, from Eq. (34), we obtain $r_c \to \sqrt{3/\Lambda}$, which agrees with what was presented in the previous section and proven in Ref. [90]. Second, if we employ Eq. (33), the equation $f(r_c, Q, \Lambda) = 0$ will be satisfied for an electric charge $Q$ of the form

$$|Q| = \frac{1}{8\sqrt{6}}\left(\frac{4\pi^4}{\alpha^4\Lambda^3} + \frac{144\pi^2}{\alpha^2\Lambda^2} + \frac{\pi(48\alpha^2\Lambda + \pi^2)^{3/2}}{\alpha^4\Lambda^3} + \frac{3\pi^3\sqrt{48\alpha^2\Lambda + \pi^2}}{\alpha^4\Lambda^3}\right)^{1/2} \neq 0. \tag{35}$$

It is clear that if we take the limit $\alpha \to \infty$, from Eq. (13), the electric charge will be $Q \to 0$, as expected.

Furthermore, since the electric charge receives a nonzero value, namely, $|Q| \neq 0$, solving equation $f(r, Q, \Lambda) = 0$ with $f(r, Q, \Lambda)$ as given by Eq. (14), we get the roots (radii of horizons)

$$r_{\pm\pm} = \pm\sqrt{\frac{3 \pm \sqrt{9 + 12Q^2\Lambda}}{2\Lambda}} \tag{36}$$

$$r_{\pm\mp} = \pm\sqrt{\frac{3 \mp \sqrt{9 + 12Q^2\Lambda}}{2\Lambda}}. \tag{37}$$

Since the radii of the horizons have to be positive, i.e., $r < 0$, the negative roots are excluded completely. The root $r_{++}$ is the largest one, so it is the cosmological horizon [92]. The root $r_{+-}$ has also to be positive, and thus it is required that $3 - \sqrt{9 + 12Q^2\Lambda} > 0$. However, this gives $Q^2 < 0$, which is impossible; thus, the root $r_{+-}$ is unphysical and is also removed. Therefore, there is no event horizon, and the singularity of the RNdS-like black hole is a naked singularity.

## IV. EFFECT OF LINEAR AND QUADRATIC GUP ON ENERGY DISTRIBUTION OF MASSLESS CHARGED PARTICLES IN RNdS-LIKE SPACETIME

Assuming that the curvature singularity can store only a few bits of information, one may say that the Hawking radiation for the black hole under study will consist of massless charged particles. Thus, it is useful to compute the total energy density of these massless charged particles.

Following the analysis in Ref. [93], we calculate the total energy density of the massless charged particles in a general spherically symmetric static spacetime in the context of linear and quadratic GUP. The first quantity to be employed for this calculation is the invariant volume element of the phase space. In Ref. [94], the invariant volume element of a phase space in a $D$-dimensional spacetime was computed in the context of linear GUP, while in Ref. [95], the invariant volume element of a phase space in $D$-dimensional spacetime was computed in the context of linear and quadratic GUP. However, in the latter case, the invariant volume was computed to $\mathcal{O}(\alpha)$, while, here we would like to be more precise and thus go up to $\mathcal{O}(\alpha^2)$. Therefore, the invariant volume element of a phase space in the context of linear and quadratic GUP to $\mathcal{O}(\alpha^2)$ is given as [96]

$$\frac{d\mathbf{x}^D d\mathbf{p}^D}{(2\pi)^D(1 - \alpha p + (\frac{2\alpha^2}{D+1} + \frac{\alpha^2}{2})p^2)^{(D+1)}}. \tag{38}$$

At the WKB level, the norm of massless particle momentum 3-space vector is

$$p^2 = p_i p^i = \frac{w^2}{f}, \tag{39}$$

where $f = f(r, Q, \Lambda)$, which is given by Eq. (14). Setting $D = 3$, the total energy density for all frequencies reads

$$\rho(f, \beta) = \gamma \int_0^\infty \frac{f^2 w^3}{2\pi^2(f - \alpha\sqrt{f}w + \alpha^2 w^2)^4} \frac{dw}{(e^{\beta w} \pm 1)}, \tag{40}$$

where $\gamma$ is the spin degeneracy, the minus stands for the massless charged bosons, and the plus stands for massless charged fermions. Upon considering the change of variable $x = \beta w/2\pi$ and $T(r) = 1/(\beta\sqrt{f})$, where $T(r)$ is the local temperature, Eq. (40) becomes

$$\rho(x, T) = 8\pi^2\gamma T^4 \int_0^\infty \frac{x^3}{(1 - ax + a^2x^2)^4} \frac{dx}{(e^{2\pi x} \pm 1)}, \tag{41}$$

where $a = 2\pi\alpha T$. This integral is not quite easy to solve. However, at least for the bosons and up to $\mathcal{O}(\alpha^3)$, it looks close to the Hurwitz zeta function

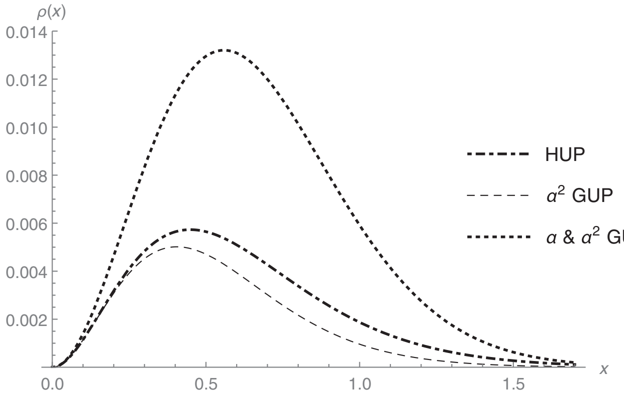


FIG. 1. The total energy density $\rho(x)$ vs the variable $x$ for bosons with $a = \frac{1}{2}$.

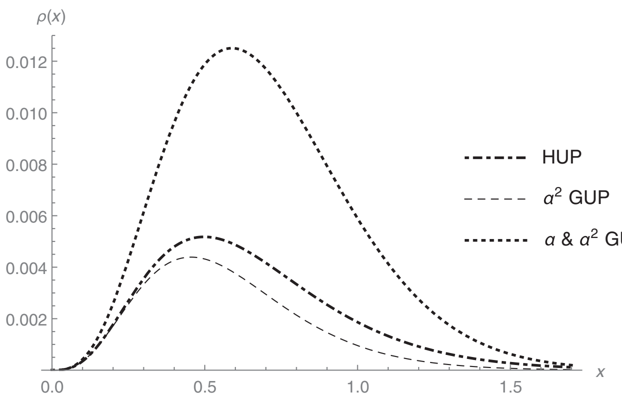


FIG. 2. The total energy density $\rho(x)$ vs the variable $x$ for fermions with $a = \frac{1}{2}$.

$$\zeta(n, u) = \frac{1}{\Gamma(n)} \int_0^\infty \frac{x^{n-1} e^{-ux}}{1 - e^x} dx, \tag{42}$$

where $a, n > 0$. In this case, the total density given by Eq. (41) is indeed a convergent integral. Thus, using contour integral techniques, Eq. (41) can be calculated. However, we focus more on demonstrating the effect of GUP on such distribution(s) and, thus, we provide Figs. 1 and 2. For fixed $\alpha$ and $T(r)$, we assume $a$ to be small compared with $x$.

A number of comments are in order here. First, for very diminutive values of $a$, as in Figs. 3 and 4, we notice that the curves of GUP tend to be that of HUP, as expected. Second, when such a collapsing system reaches the state of ultracold black hole, where $T(r) \sim \kappa_c \to 0$ [97], and since the constant $a$ is small only if $\alpha$ is also small, it is evident that we have the GUP to tend to the HUP, as expected. Third, if we keep the GUP parameter $\alpha$ fixed and consider a different metric component $f \sim 1/T^2(r)$, say, for instance, the $f(M, r, Q, \Lambda)$ of the standard massive RNdS spacetime, to be compared with the larger $f(r, Q, \Lambda)$ of the massless

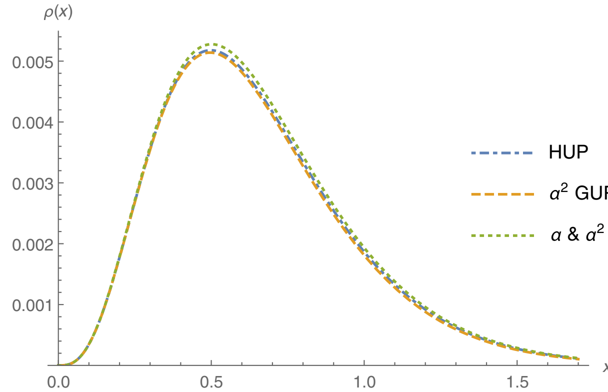


FIG. 3. The total energy density $\rho(x)$ vs the variable $x$ for bosons with $a = 0.01$.

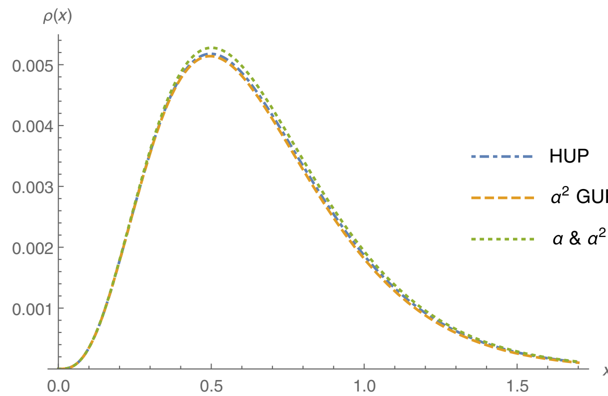


FIG. 4. The total energy density $\rho(x)$ vs the variable $x$ for fermions with $a = 0.01$.

RNdS spacetime for any allowed physical radius, then we notice that the massless one is colder, and hence $a$ is smaller for the massless RNdS spacetime, as expected.

## V. CONCLUSIONS

In this work, we have followed the methodology presented in Refs. [89,90], except we have introduced a linear term in momentum in the GUP, namely, linear and quadratic GUP, in order to investigate the GUP effect on a black hole system. We first computed the GUP-modified temperature and, using the first law of black hole mechanics, the GUP-modified entropy of the cosmological horizon. In our case, the entropy does not only have a quadratic GUP correction term but also a linear GUP correction term. Then, for the specific massless RNdS spacetime, since the modified entropy of the cosmological horizon depends explicitly on the cosmological radius, we calculated the GUP-modified cosmological radius. Moreover, we expressed the electric charge of the specific black hole in terms of the GUP-modified cosmological horizon radius. Since the electric charge is nonzero, the equation for the locations of the black hole horizons is solved.

The cosmological horizon is the only physical horizon, and thus it exists while there is no event horizon. Therefore, the singularity, i.e., $r = 0$, is a naked singularity, and thus the cosmic censorship conjecture is violated in the black hole spacetime under study. Furthermore, in Refs. [89,90], it was also shown that, by considering the quadratic GUP, the second law of black hole mechanics prevents the occurrence of massless charged particles. Assuming that the singularity can store only a few bits of information, one may say that the Hawking radiation for the black hole under study will consist of massless charged particles. For this reason, we also compute the total energy density of these massless charged particles in RNdS-like spacetime and in the presence of the linear and quadratic GUP. Our result does not say that massless charged particles can exist within the contemporary known Standard Model. Rather, contrary to Ref. [90], it says that, until we get a concrete theory of quantum gravity, there is no physical principle that prohibits the existence of massless charged particles upon combining the second law of black hole mechanics together with the more general, linear, and quadratic GUP. It is also worth noting that if massless charged particles existed in low/moderate energies they would have been detected easily.

In contrast, this is not contradictory to the most famous hypothesis for massless charged particles in which massless quarks are believed to exist at very high energies before symmetry breaking occurred (see, for instance, Ref. [98–100]). Despite that we discussed an *ultracold* black hole–like system, we showed that in the presence of linear and quadratic GUP the corresponding massless charged particles have huge energy density compared with those of HUP and of the quadratic GUP. Even if those tentatively assumed massless charged particles are indeed not comparable to massless quarks in their features or the way of formation, the gigantic effect of gravitational collapse near the *fundamental length*, that is necessary to form massless charged particles, might be comparable to the high-energy condition to form massless quarks.

Finally, we would like to emphasize the subtlety of both fundamental topics, namely, the cosmic censorship conjecture and the massless charged particles, discussed here. It is our belief that the fundamental topics can not be resolved using phenomenological, semiclassical, and/or heuristic methodologies of quantum gravity [101]. Thus, we agree with Xiang and Shen on the indispensability of a full theory of quantum gravity theory to be applied when a star collapses in order to get the required full picture of such phenomena.


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