A Chiral Icosahedral QC and its Mapping to an E8 QC

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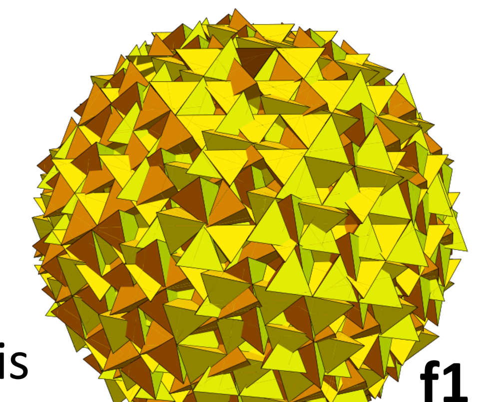
Introduction The Fibonacci Icosagrid (FIG) QC is introduced along with an unexpected mapping to a Golden Ratio based composition of 3D slices of Elser and Sloane's 4D QC projected from E8. Because E8 encodes all gauge symmetry transformations between particles and forces of the standard model of particle physics and gravity, this novel QC may be useful for a loop quantum gravity type approach to unification physics.

Fibonacci IcosaGrid (FIG)

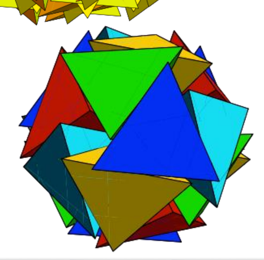
IcosaGrid and FCC

The 3D IcosaGrid, shown in **f1**, is made of 10 sets of equidistant planes that are parallel to the facets of the icosahedron.



f1

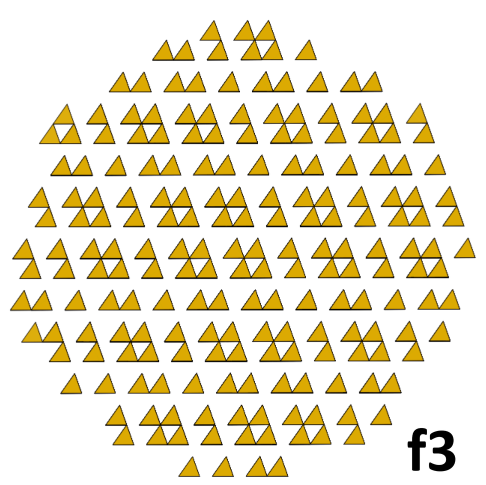
An alternative way of looking at the IcosaGrid is as five FCC lattices rotated in relation to one another with a Golden Angle [1] of $Cos^{-1}\left(\frac{3\phi - 1}{4}\right)$, where Φ is the Golden Ratio. The core is a chiral 20-tetrahedron cluster (20G) shown in **f2**.



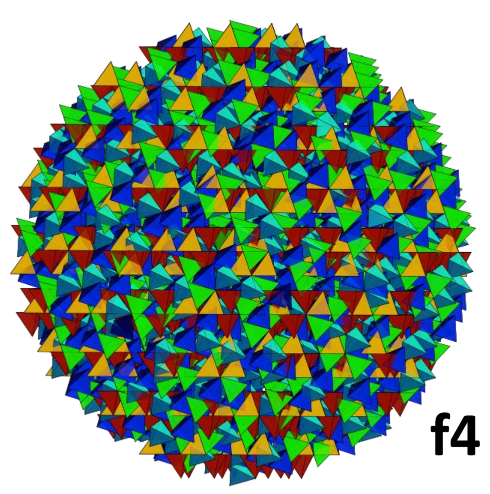
f2

FIG QC derived from IcosaGrid

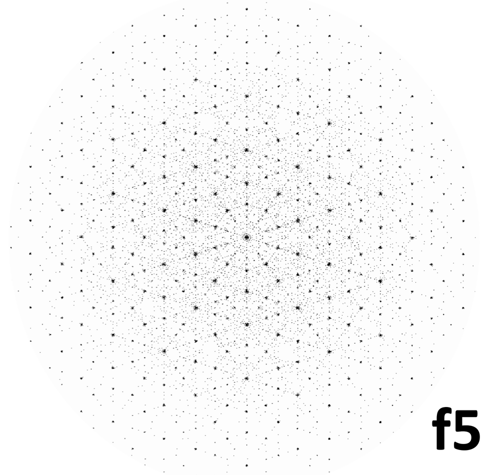
The structure in **f1** is not a QC due to the arbitrary closeness of its points. By modifying the spacing between parallel planes in each FCC set to be the Fibonacci chain (with 1 and Φ elements) – see **f3**, the structure becomes a QC (**f4**). The diffraction pattern is shown in **f5**.



f3



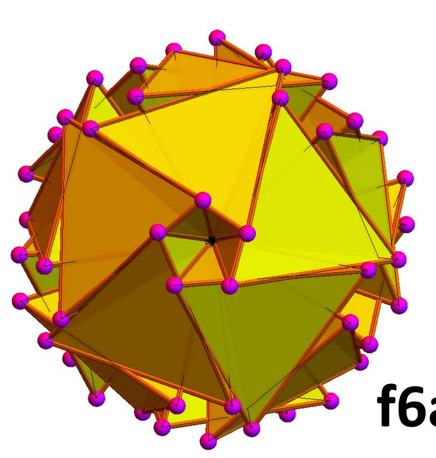
f4



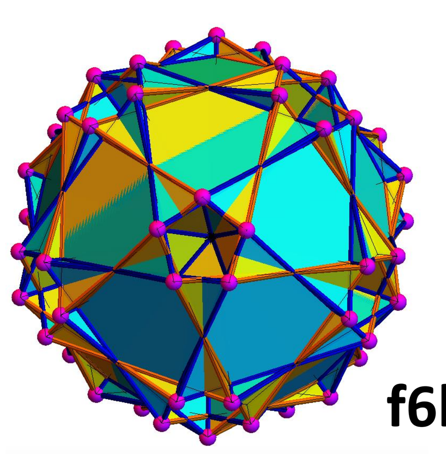
f5

20G, 600-cell compound and their Golden Angle

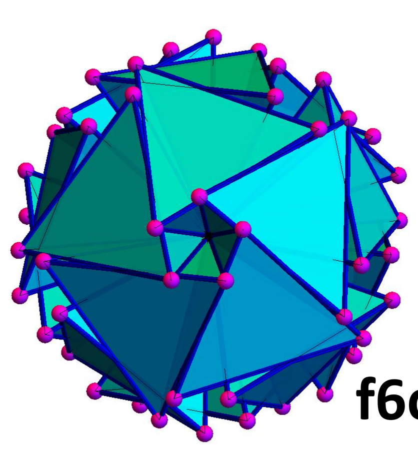
The center of the FIG is also a 20G (**f2**). Right an left chiralities share the same point set (**f6a-c**). The core of the ESQC is a 600-cell. A 120 cell is a compound of five 600-cells rotated from one another by the Golden Angle (**f7a-c** shows a 3D projection of part of the connection). It has the same chiral properties as the 20G. A deep relationship exists between these three structures: (1) FIG, (2) CQC, (3) these 4D Platonic solids, the 600-cell and its five compound.



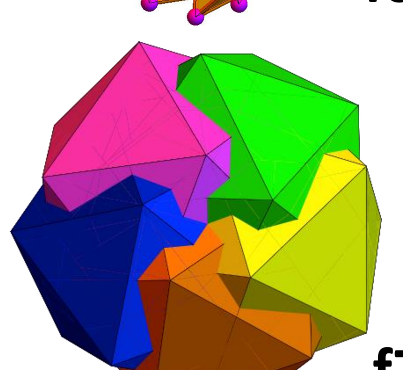
f6a



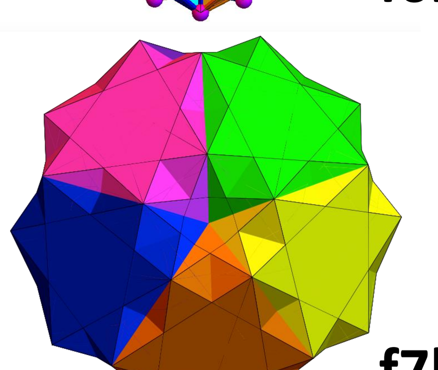
f6b



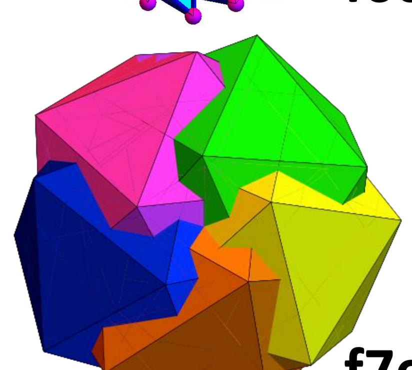
f6c



f7a



f7b



f7c

The E8 QCs

ESQC

The Elser-Sloane QC (ESQC)[2] is a 4D QC obtained as a projection of the 8D lattice E8. It is a highly symmetric ([3,3,5]) QC made of intersecting full or partial 600-cells. The projection mapping matrix is shown on the right:

Projection mapping matrix:

$$\Pi = -\frac{1}{\sqrt{5}}\begin{bmatrix} \phi^{-1}I & H \\ H & \phi I \end{bmatrix},$$

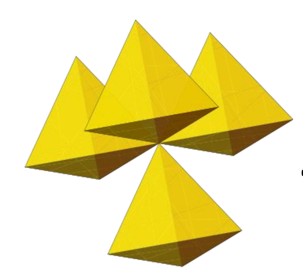
where $I = I_4 = \mathrm{diag}\{1,1,1,1\}$,

$$\phi = \frac{\sqrt{5}-1}{2},\ \mathrm{H} = \frac{1}{2}\begin{bmatrix} -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix}.$$

CQC

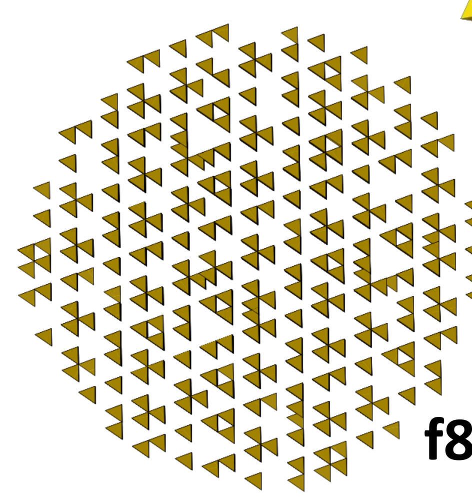
ESQC has two cross-sections that are 3D QCs with tetrahedral symmetry. The first (Type I, shown in **f8**) has larger tetrahedra. And the second (Type II, shown in **f9**) has smaller tetrahedra that are 1/Φ the length of Type . Type I has four tetrahedra at its center (**f10**), while Type II has only one at its center.

Five copies of Type I, rotated from one another by the Golden Angle, generate an icosahedrally symmetric Compound QC (CQC) (**f11**). And 20 copies of Type II rotated by the same angle make a similar but sparser CQC (**f12**). They share the same core structure as the FIG, the 20G shown in **f2**.

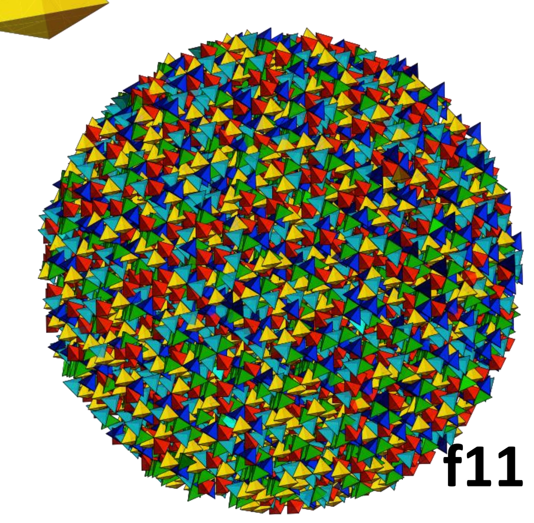


f10

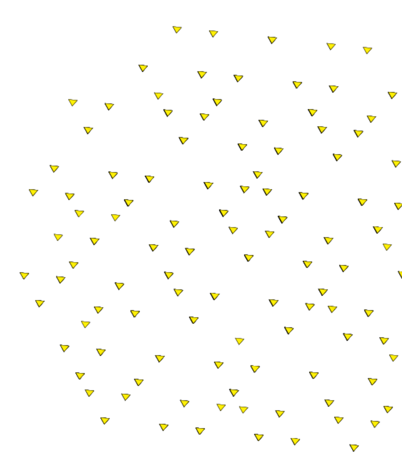
$Cos^{-1}\left(\frac{3\phi - 1}{4}\right)$



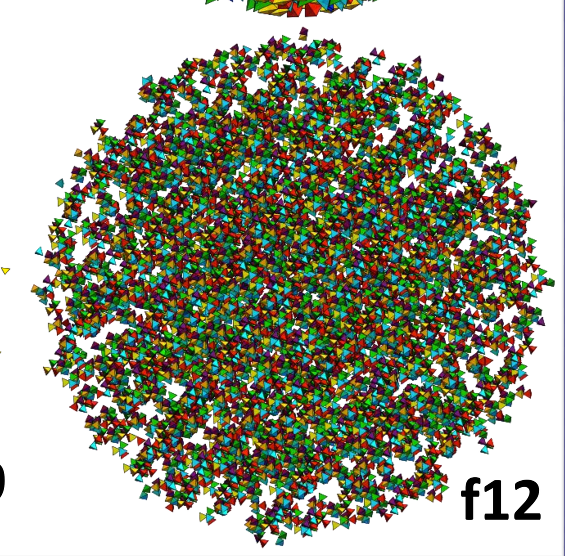
f8



f11



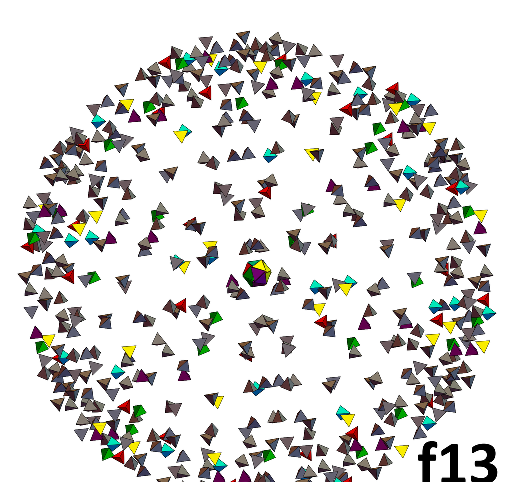
f9



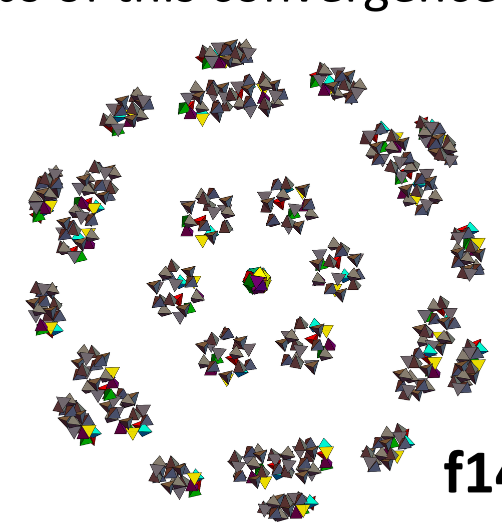
f12

Converging of the 20Gs with the Golden Rotation

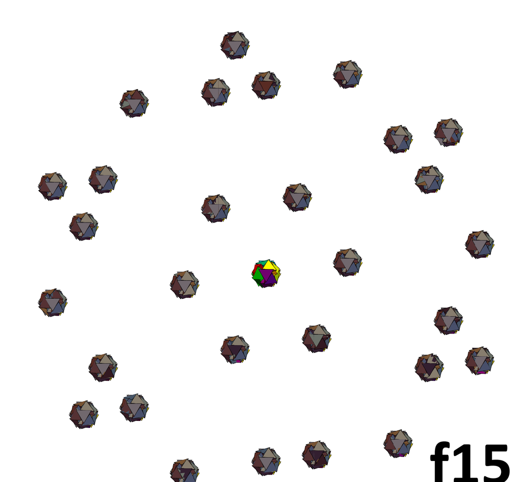
Why use the Golden Angle for composing the CQC? Besides creating a deep connection between the FIG and ESQC, it converges the 5 or 20 slices of the ESQC into a perfect non-crashing QC. The below frames show a few steps of this convergence:



f13



f14

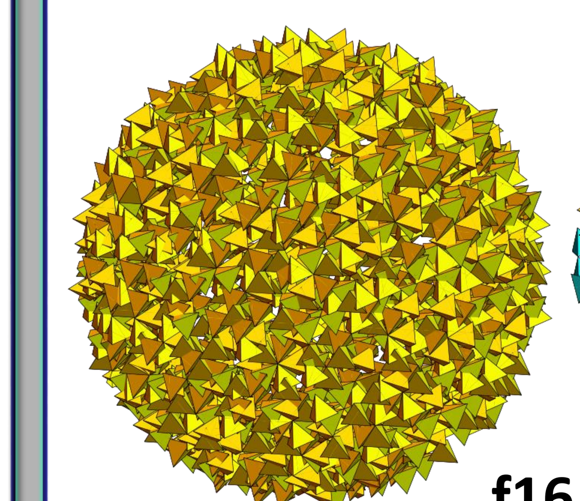


f15

Mapping between FIG and CQCs

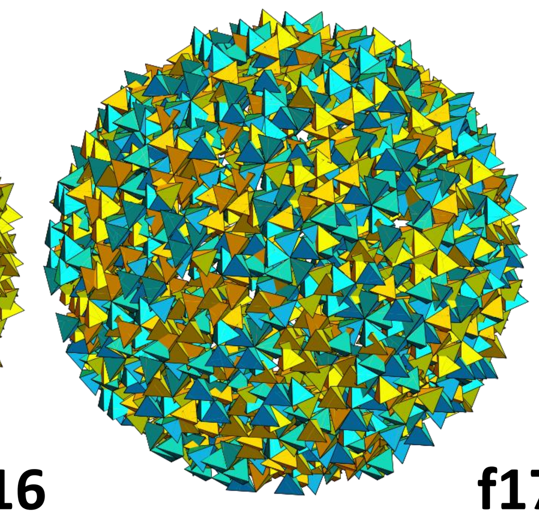
The FIG QC and CQC are built in completely different ways. Yet, to our surprise, the FIG QC completely embeds the Type I and Type II CQCs. And the Type I CQC also completely embeds in Type II. All tetrahedra shown in **f16-18** are members of the FIG QC. The cyan and red tetrahedra are the CQCs and are subsets of the FIG. The key to this perfect mapping is the Fibonacci chain modification of the FCC lattices (**f3, f8, f9**) and the Golden Ratio Rotation.

FIG



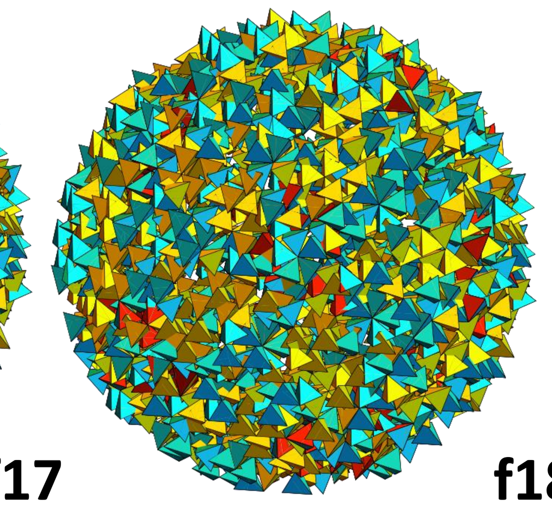
f16

FIG + Type I CQC



f17

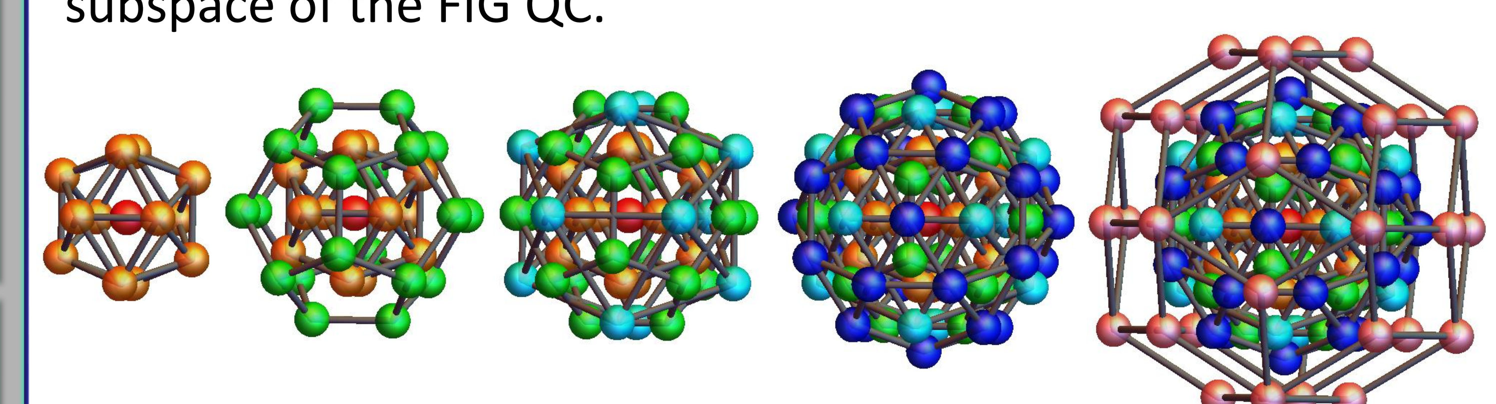
FIG + both CQCs



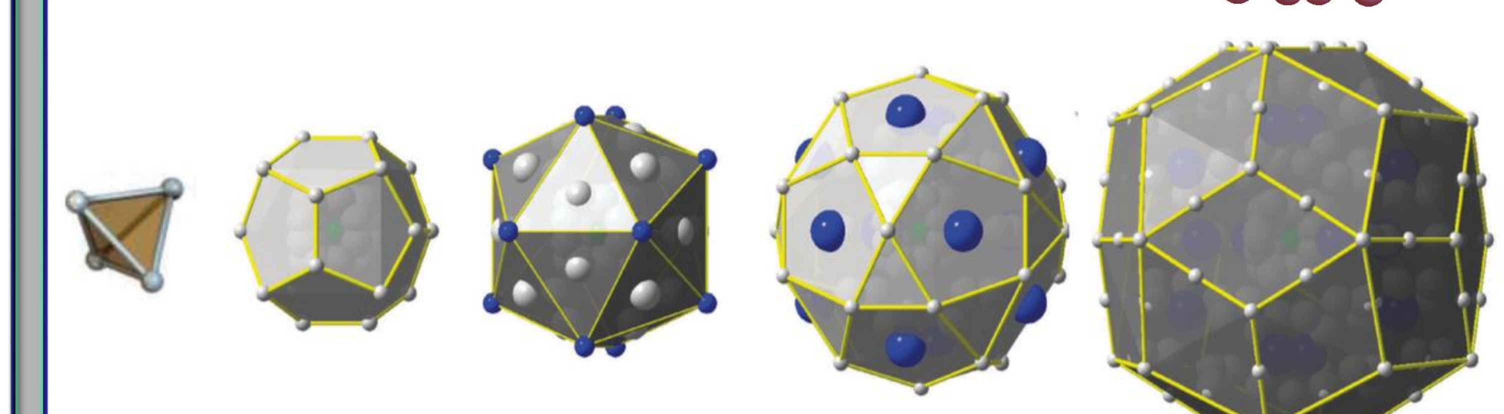
f18

Mapping between ESQC, Tsai-type QC and FIG

Projecting the center (600-cell) of the ESQC to 3D (**f19**) generates the point set of a Tsai-type QC (**f20**) [3], except for the difference in the first layer. However, the full permutation cycle of the center (tetrahedron) Tsai-type QC might form an icosahedrally symmetric pattern. We suspect that this permutation set of the Tsai-type QC relates each tetrahedral position to the others by the Golden Ratio Rotation in the FIG. Accordingly, the Tsai-type QC could be a subspace of the FIG QC.



f19



f20

References

1. F. Fang, K. Irwin, J. Kovacs and G. Sadler, *arXiv:1304.1771* (2013).
2. V. Elser and N.J.A. Sloane, *J. Phys. A*, **20**, 6161-6168 (1987).
3. M. de Boissieu, *Chem. Soc. Rev.*, 2012, 41, 6778–6786 (2012).