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The Cycloidal Fractal Signatures in Fibonacci Chain

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Outline

- **Introduction of the Fibonacci chain**
- **The fractal signature of the Fibonacci chain in Fourier space**
  - ❑ The fundamental Cycloidal Fibonacci fractal and its pointwise dimension
  - ❑ The derivatives of the Cycloidal Fibonacci fractal
- Summary

Fibonacci chain

Substitution rule: L -> LS and S -> L, where S and L represents short and long segments in the 1D quasicrystal, where $\frac{L}{S} = \varphi$.

# of iterations	Fibonacci chain	Length	Geometric representation
1	S	1	
2	L	1	
3	LS	2	
4	LSL	3	
5	LSLLS	5	
6	LSLLSLSL	8	
7	LSLLSLSLLSLLS	13	
8	LSLLSLSLLSLLSLSLLSLSL	21	
..	...	...	

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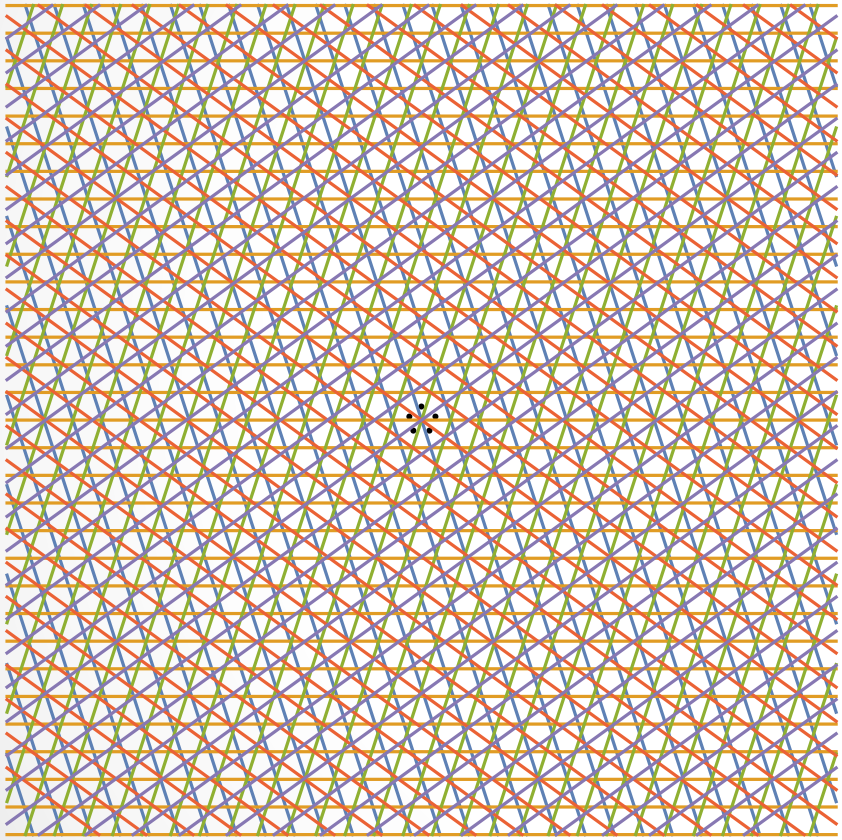
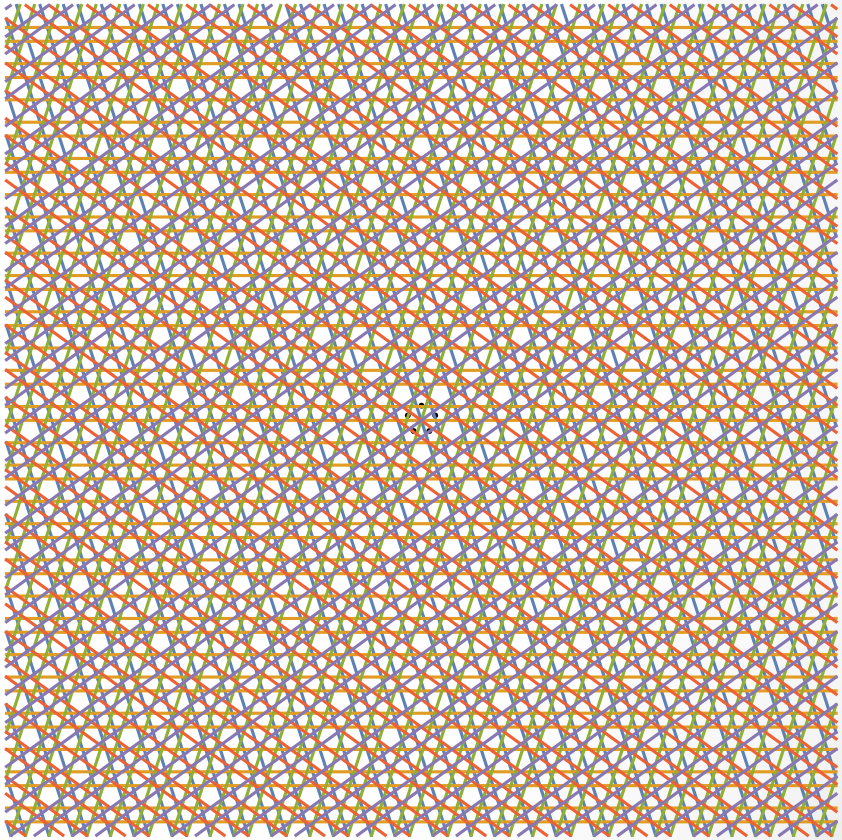
Fibonacci chain can convert a non-quasicrystalline grid space into a QC

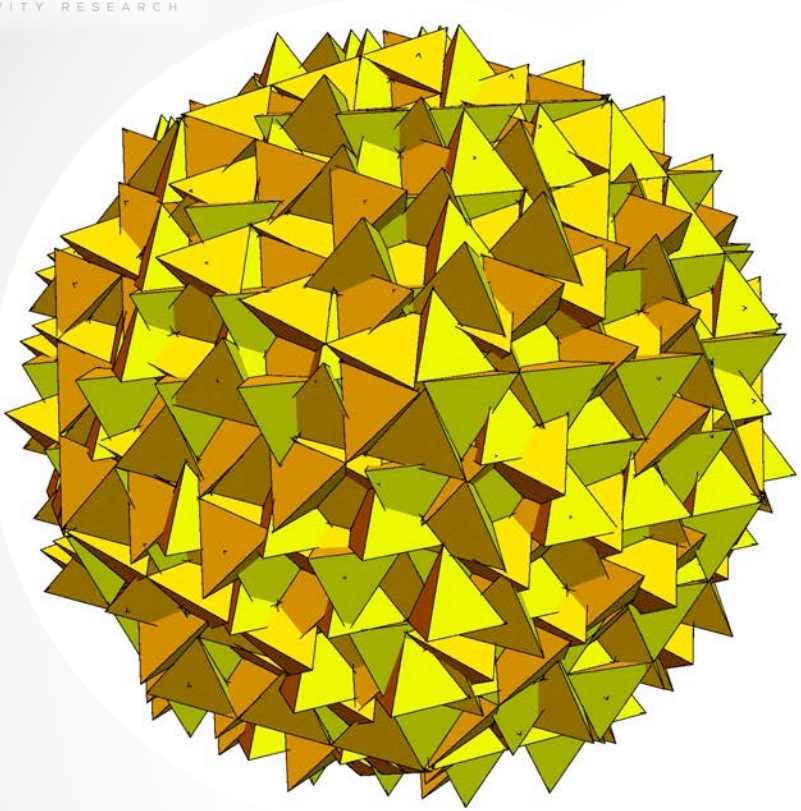


Pentagrid

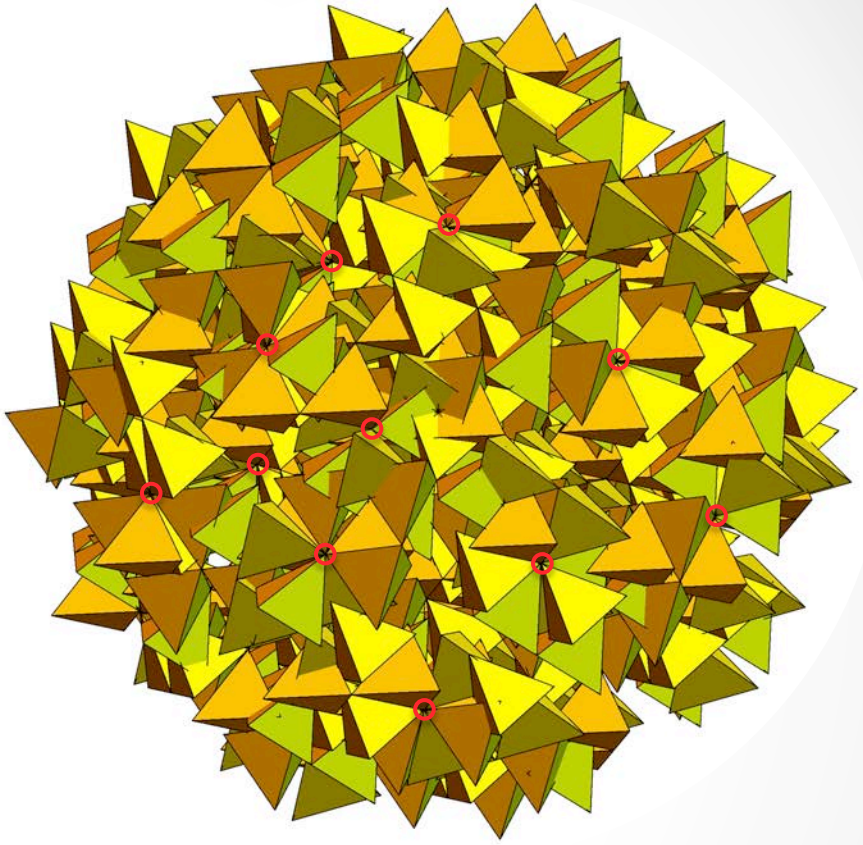


Fibonacci Pentagrid

Fibonacci chain and 3D grids



Icosagrid



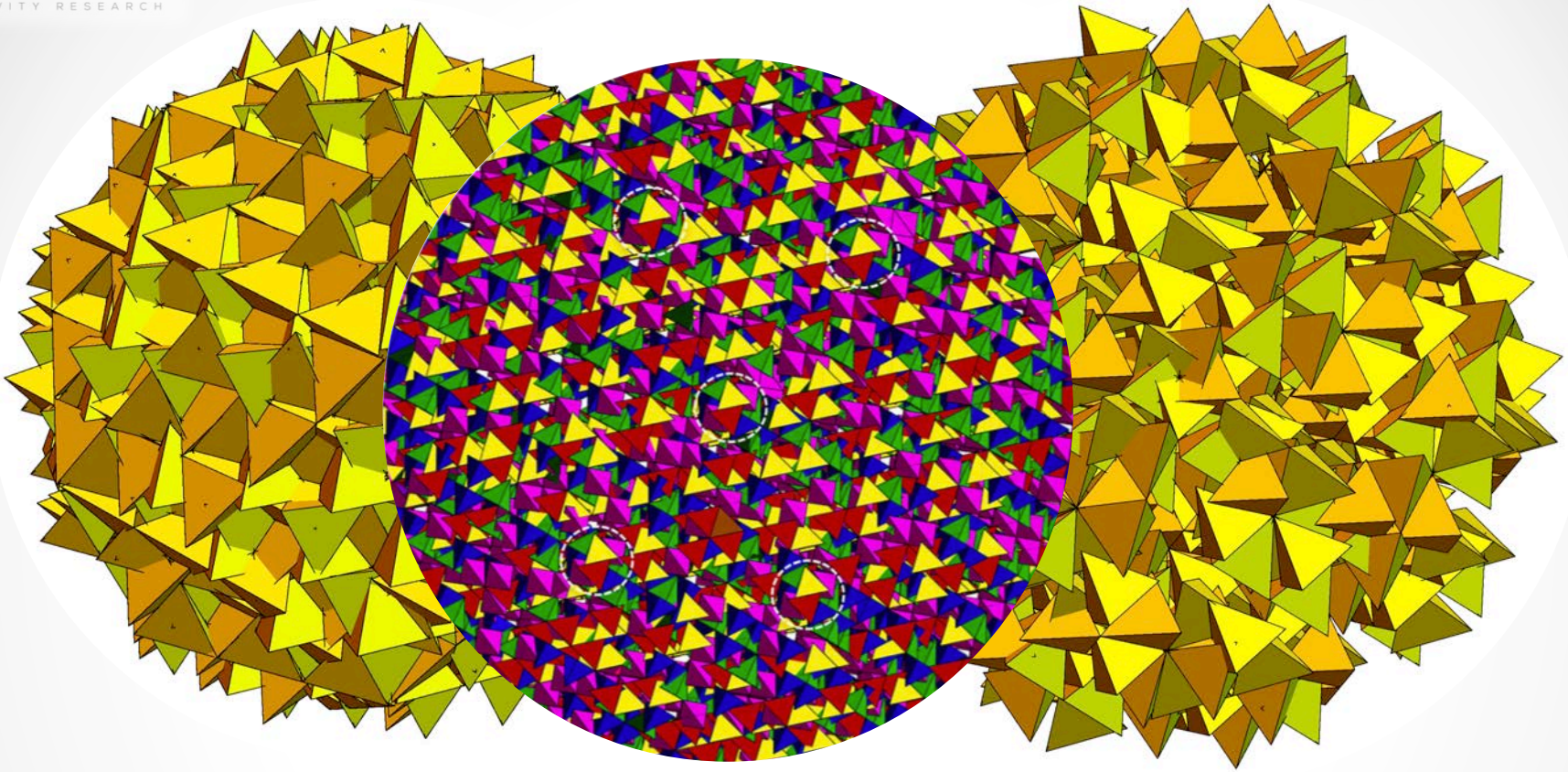
Fibonacci Icosagrid

An Icosahedral Quasicrystal and E_8 derived quasicrystals

F. Fang* and K. Irwin

Quantum Gravity Research, Los Angeles, CA, U.S.A.

Fibonacci chain and 3D grids



Icosagrid

Fibonacci Icosagrid

An Icosahedral Quasicrystal and E_8 derived quasicrystals

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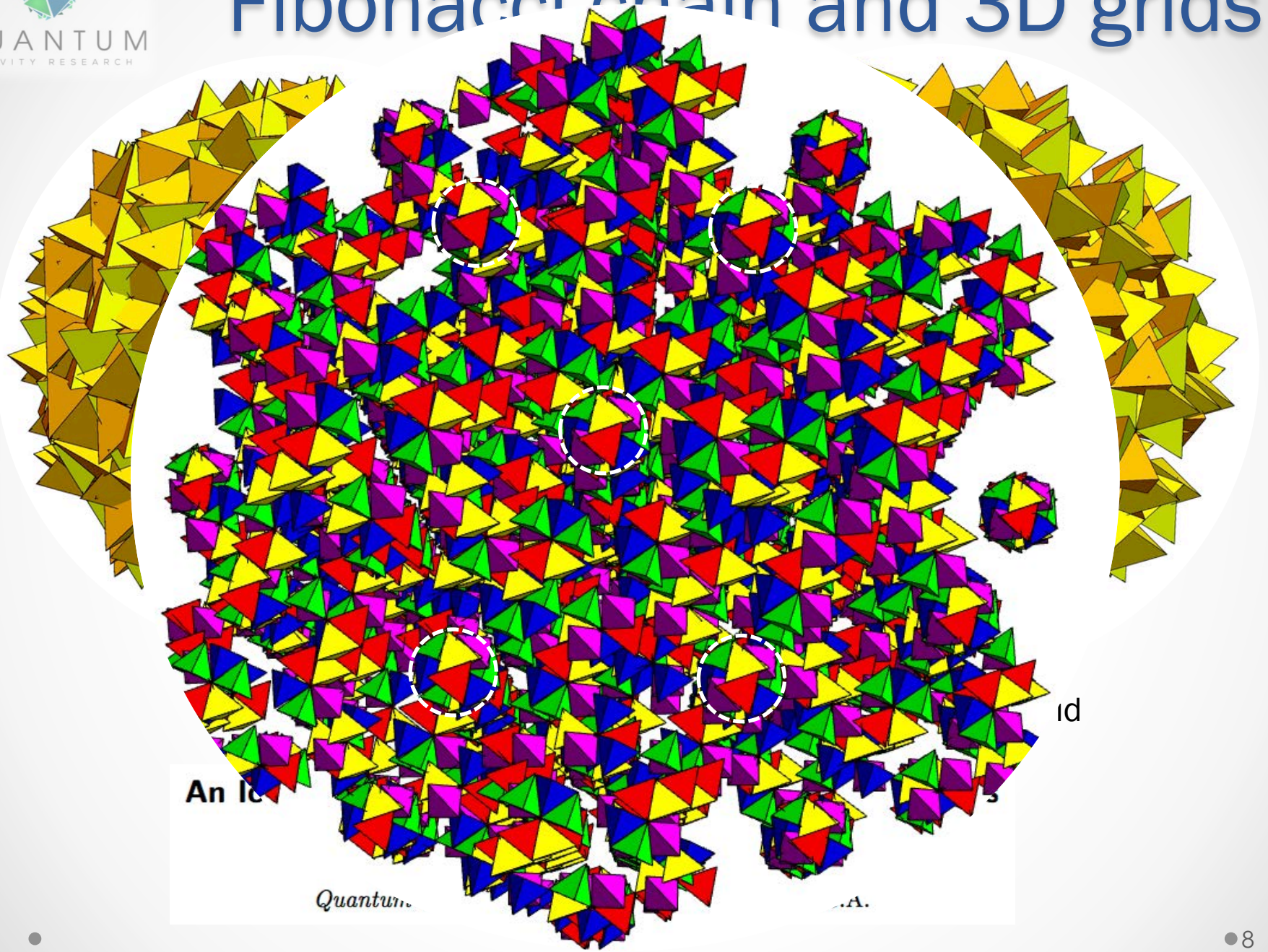
Fibonacci chain and 3D grids

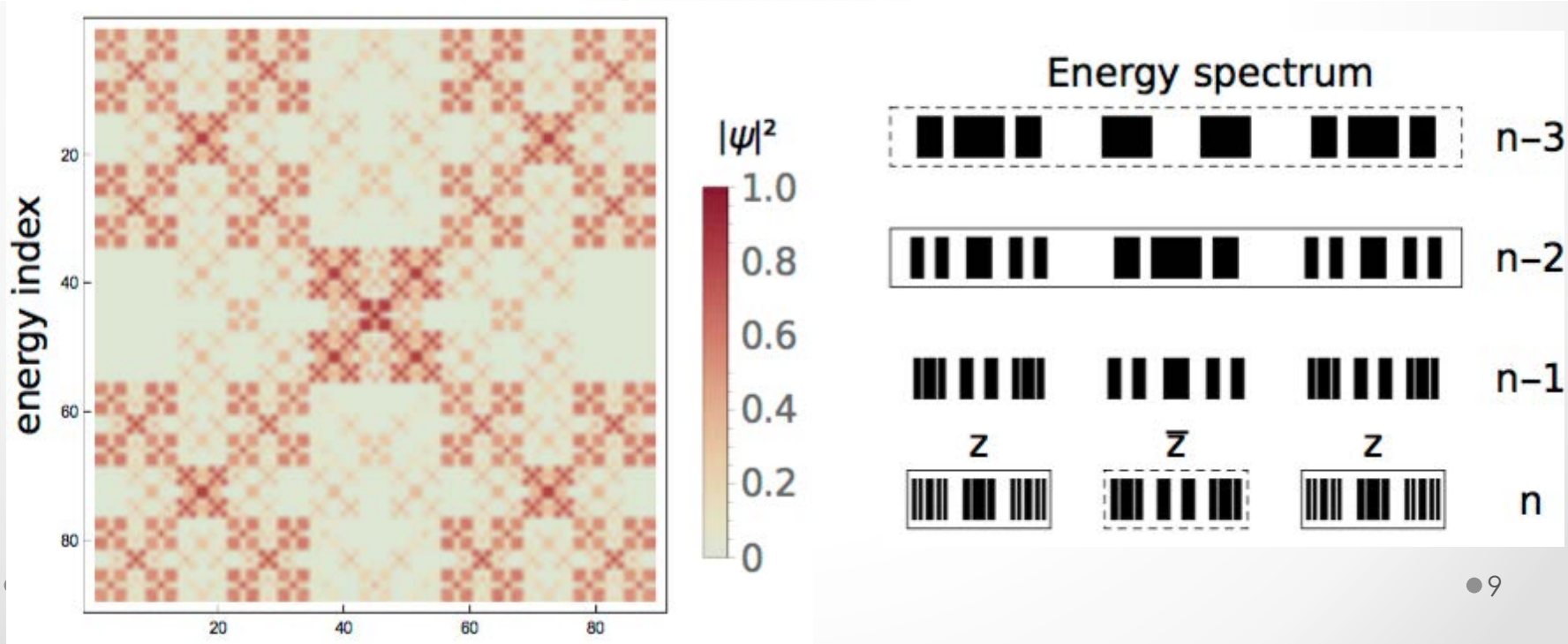


Fractal properties of Fibonacci chain

Quasicrystals – *long-range order and self similarity* – Fractal

Fractal dimensions of the wavefunctions and local spectral measures on the Fibonacci chain.

Nicolas Macé,[1] Anuradha Jagannathan,[1] and Frédéric Piéchon[1]
[1] *Laboratoire de physique des Solides, Université Paris-Saclay, 91400 Orsay, France*
(Dated: January 5, 2016)



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Fractal properties of Penrose tiling

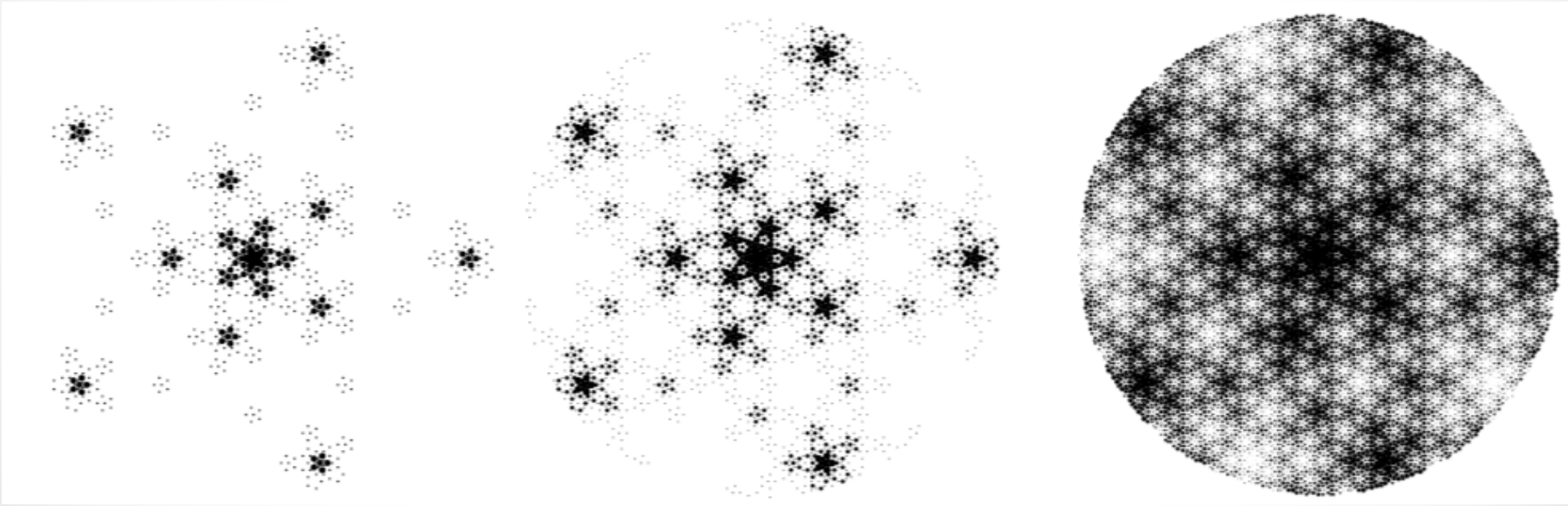
0.1 Energy spectra and eigenstates of quasiperiodic tight-binding Hamiltonians

Uwe Grimm

Applied Mathematics Department, Faculty of Mathematics and Computing,
The Open University, Walton Hall, Milton Keynes MK7 6AA, United Kingdom

Exact Eigenstates of Tight-Binding Hamiltonians on the Penrose Tiling

Przemysław Repetowicz, Uwe Grimm, and Michael Schreiber
Institut für Physik, Technische Universität Chemnitz, D-09107 Chemnitz, Germany
(February 7, 2008)



Increasing the amplitude of the wave function

Fractal signature of the Fibonacci chain in Fourier space

$$Z_s = \frac{1}{\sqrt{n}} \sum_{r=1}^{n} u_r \, e^{2\pi i \, (r-1) \, (s-1)/n}$$

$$u_r = (2 - \phi) + (\lfloor (r+1)\,\phi \rfloor - \lfloor r\,\phi \rfloor) \, (\phi - 1)$$

where Φ is the golden ratio and u_r defines the Fibonacci chain segments, L or S, here Φ or 1.

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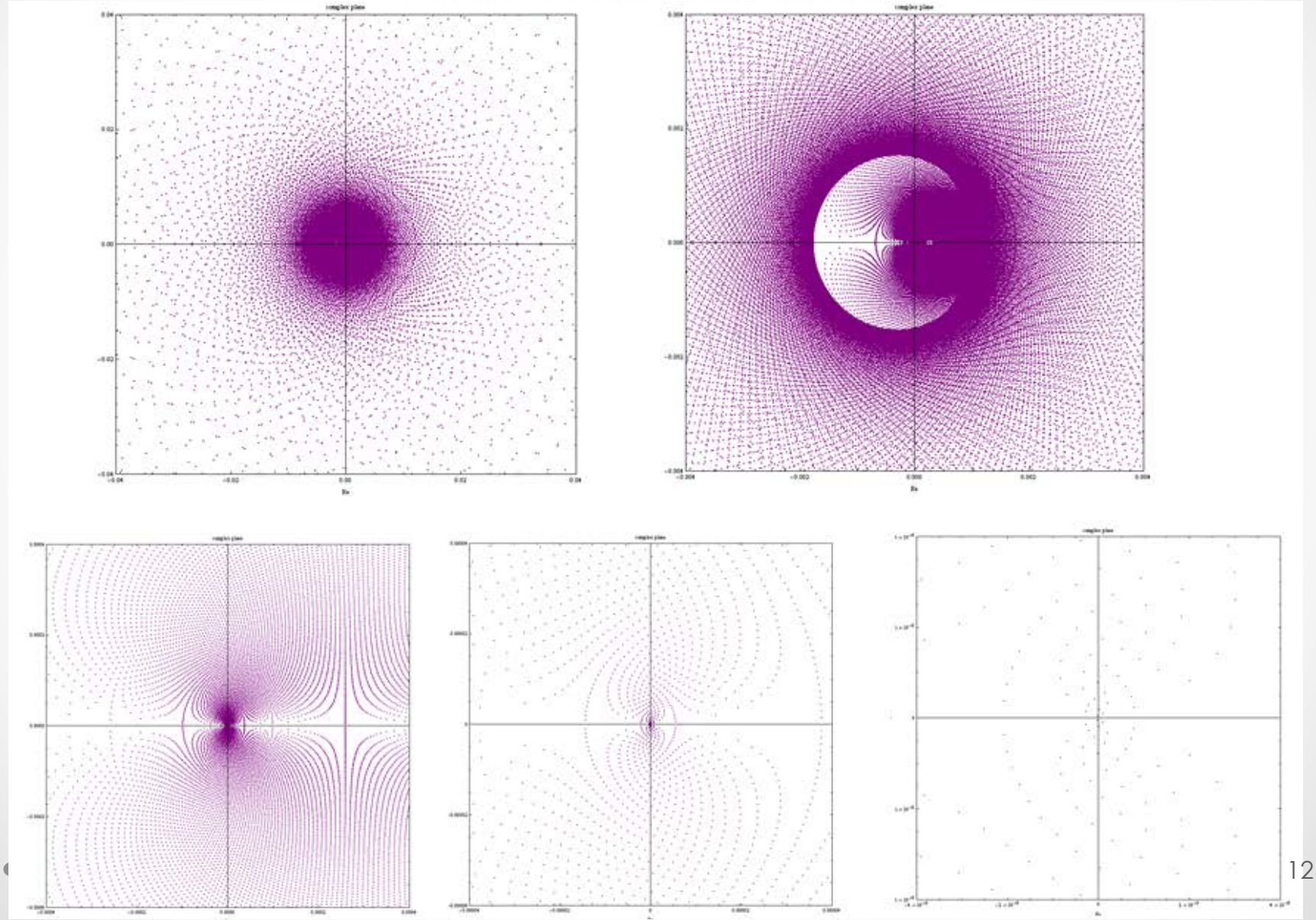
The fractal signature of the Fibonacci chain in Fourier space



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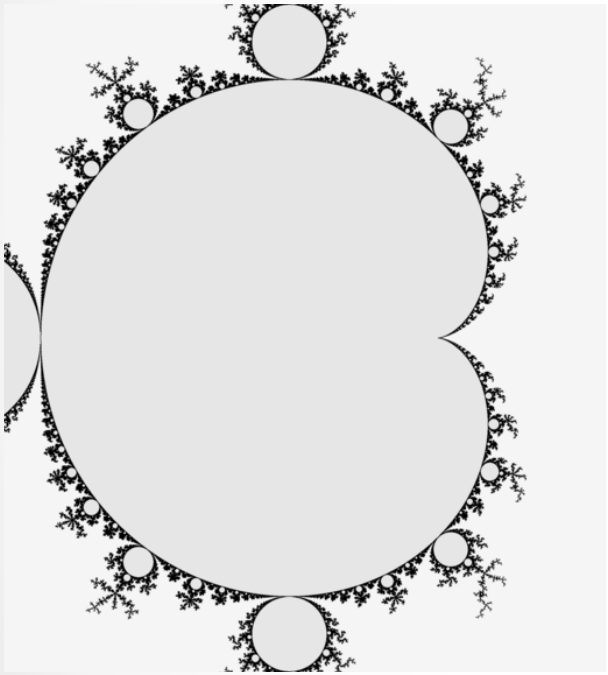
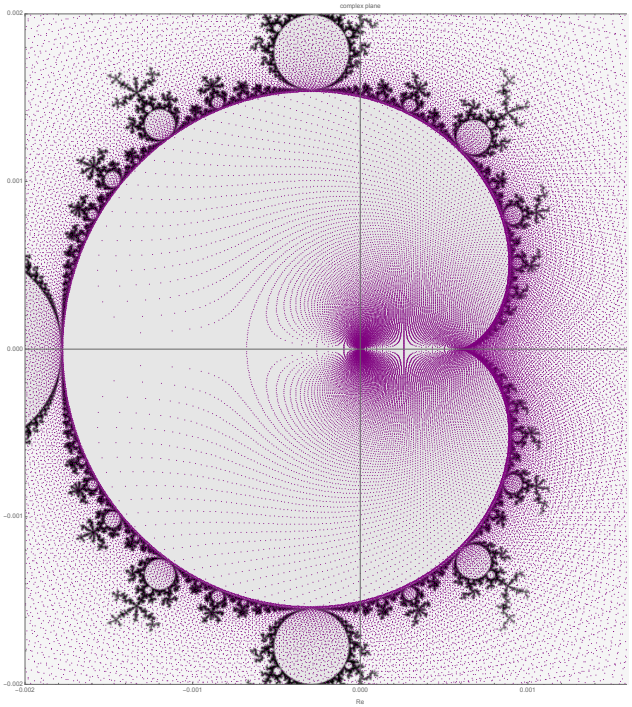
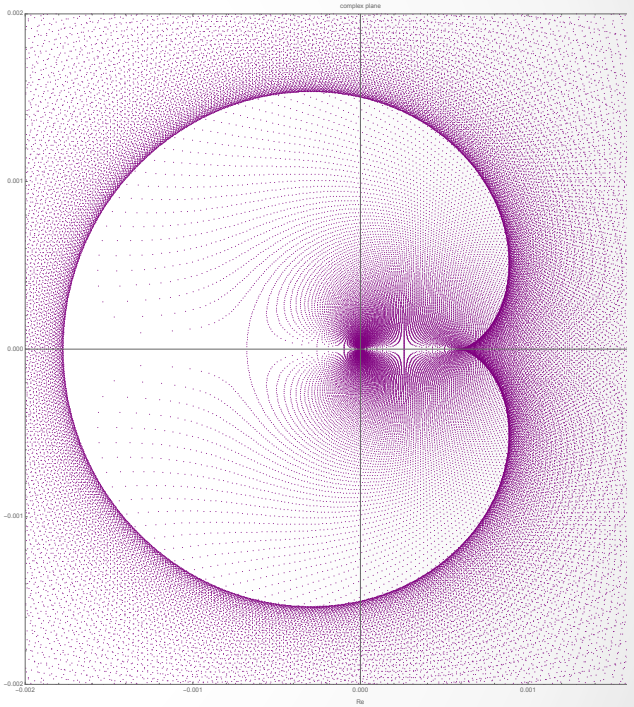
Comparison with the Mandelbrot set



a



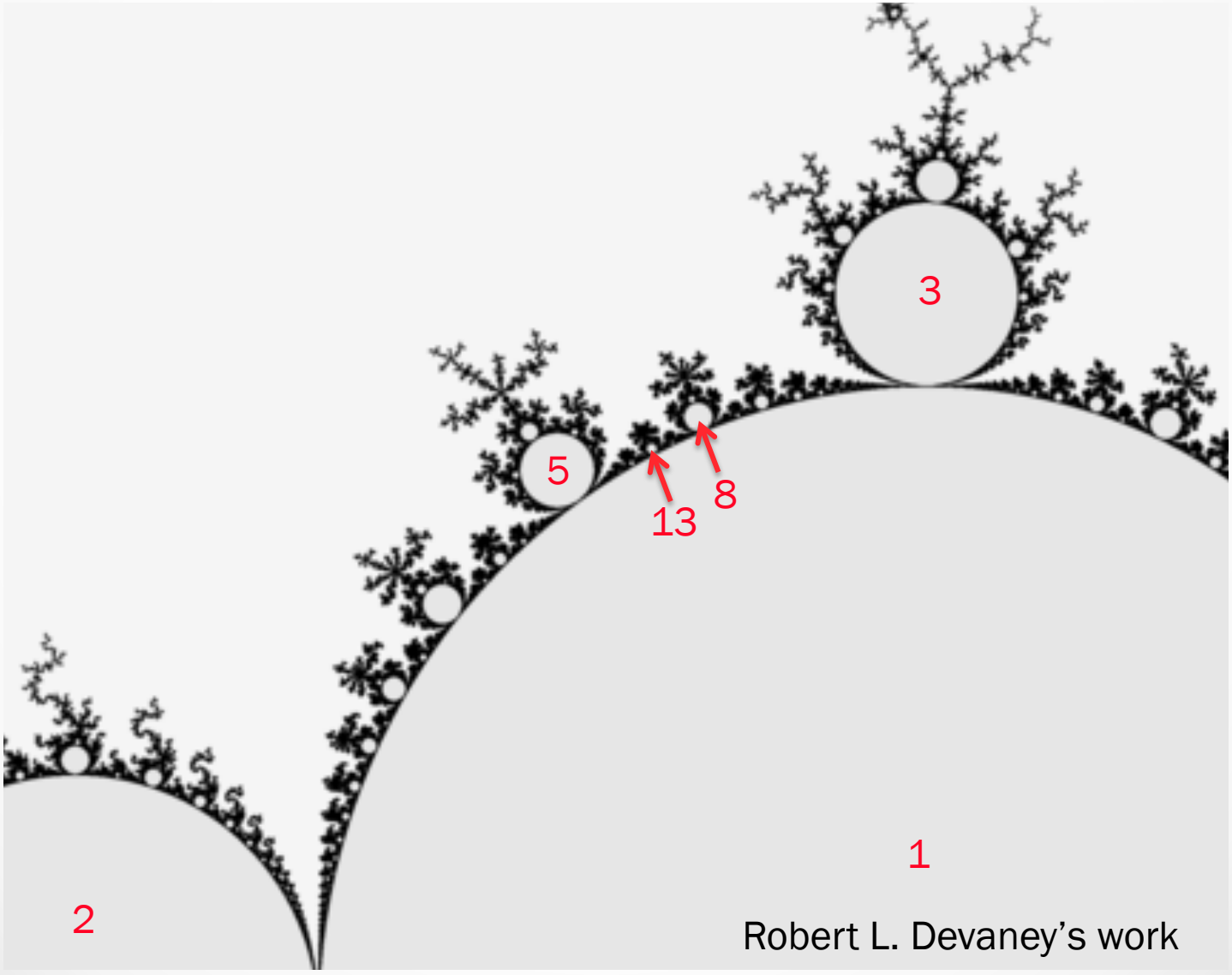
b



c

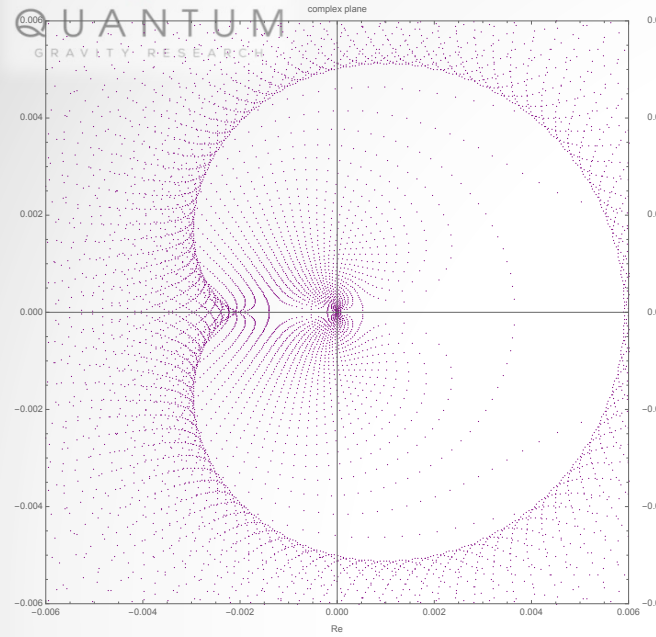
Mandelbrot set picture courtesy of Adam Majewski – from Wikipedia page

Mandelbrot set and Fibonacci sequence

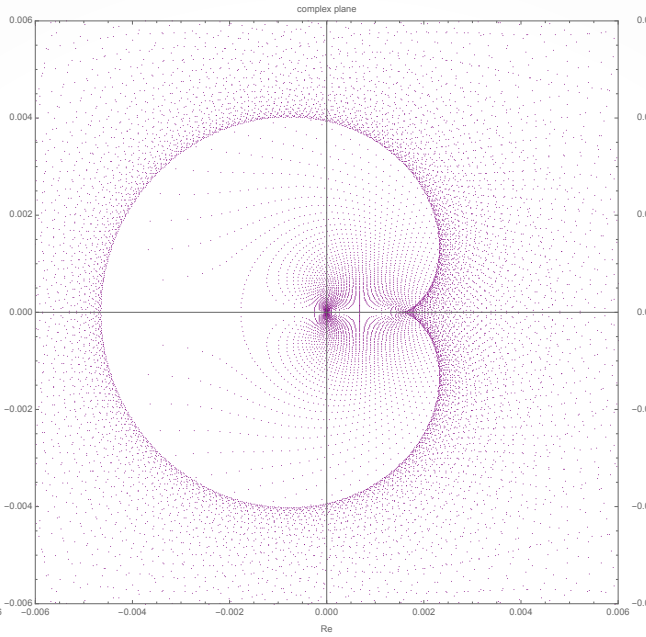


Robert L. Devaney's work

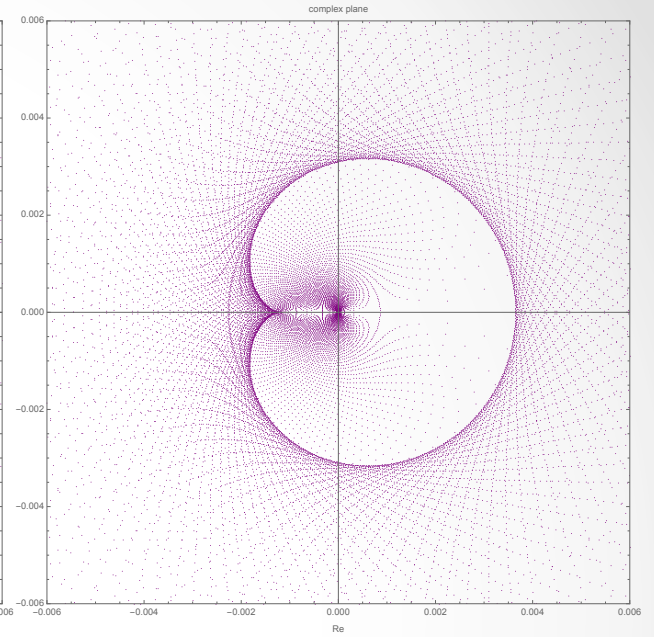
20 iterations



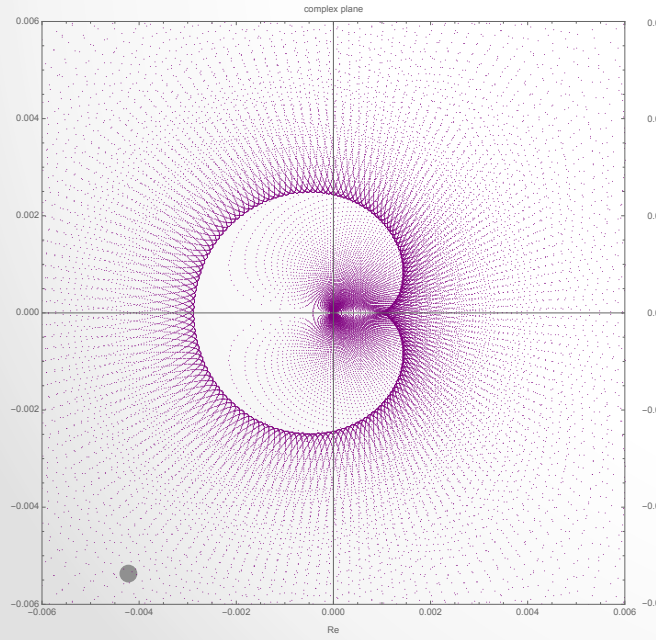
21 iterations



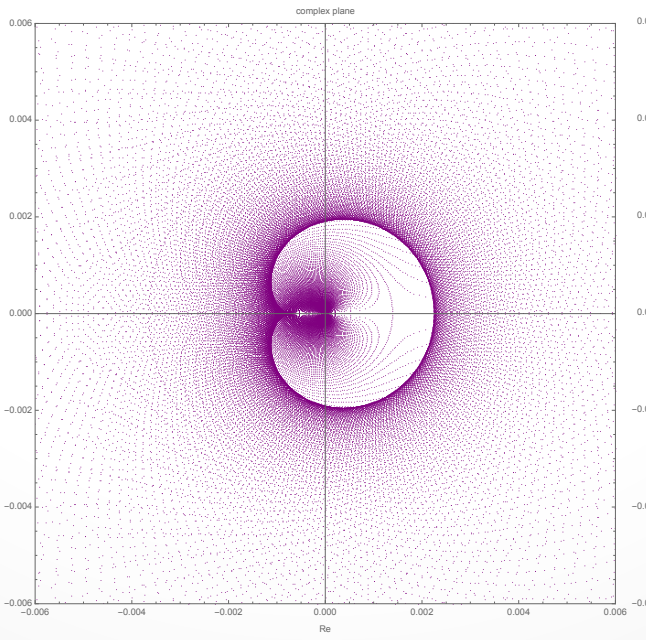
22 iterations



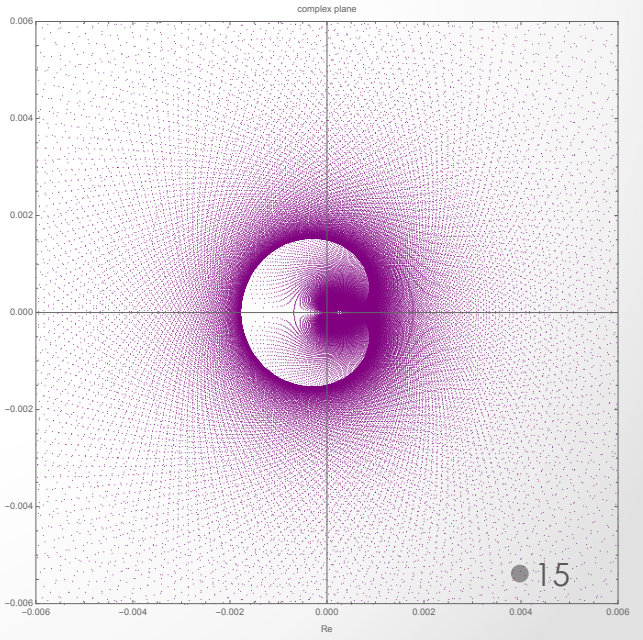
23 iterations



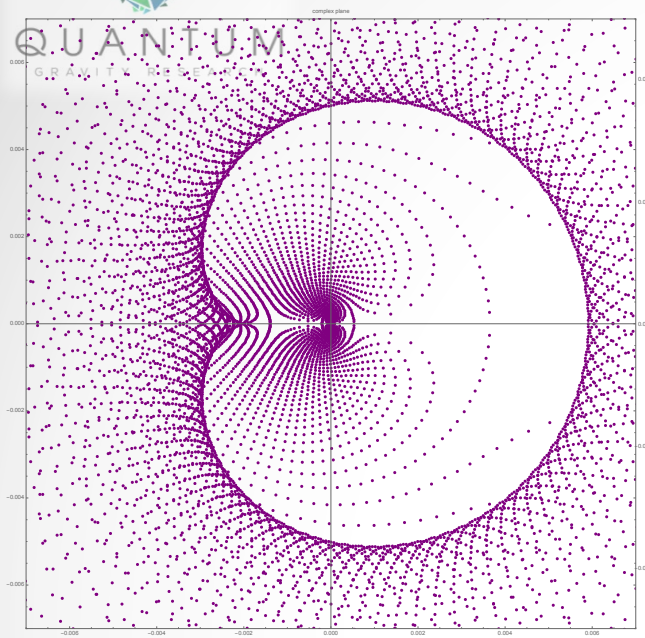
24 iterations



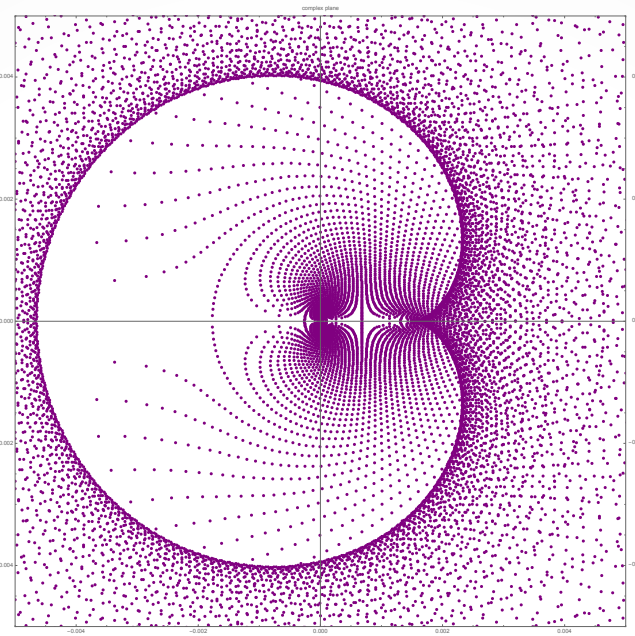
25 iterations



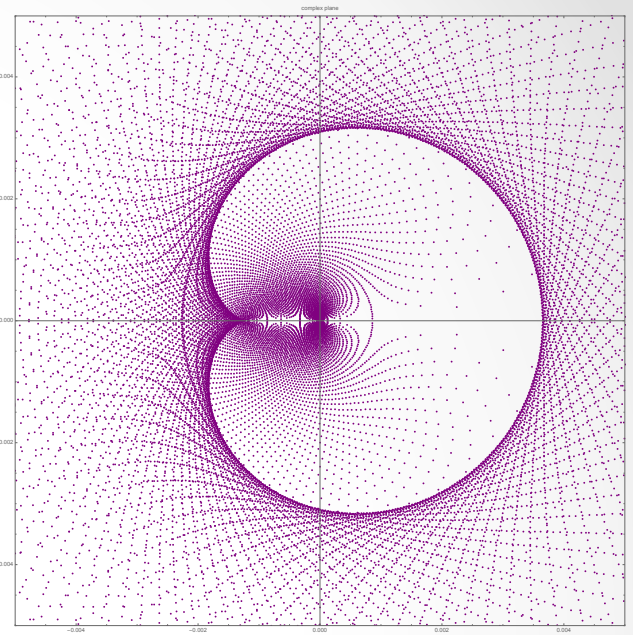
20 iterations



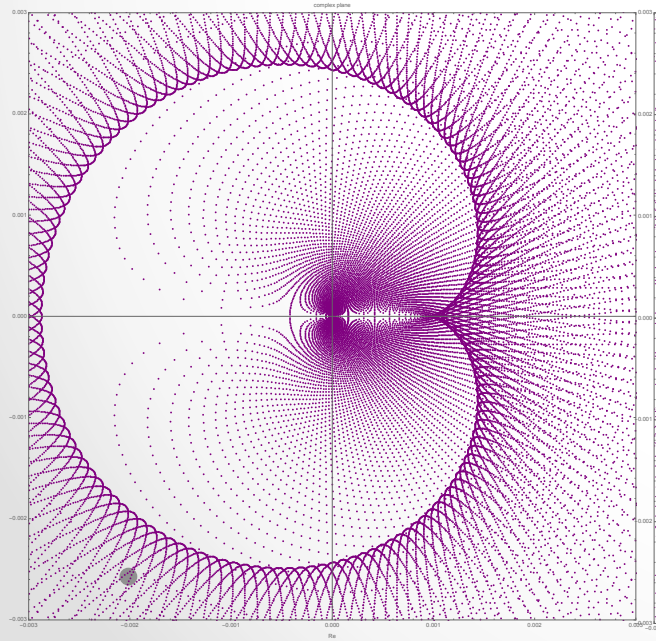
21 iterations



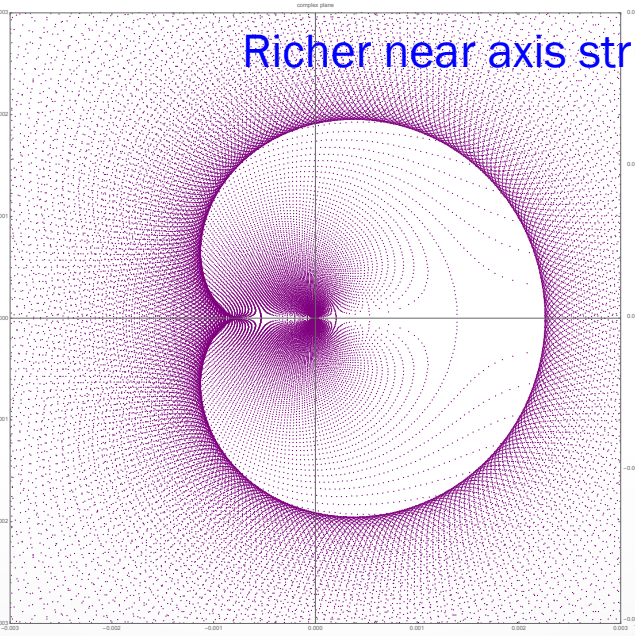
22 iterations



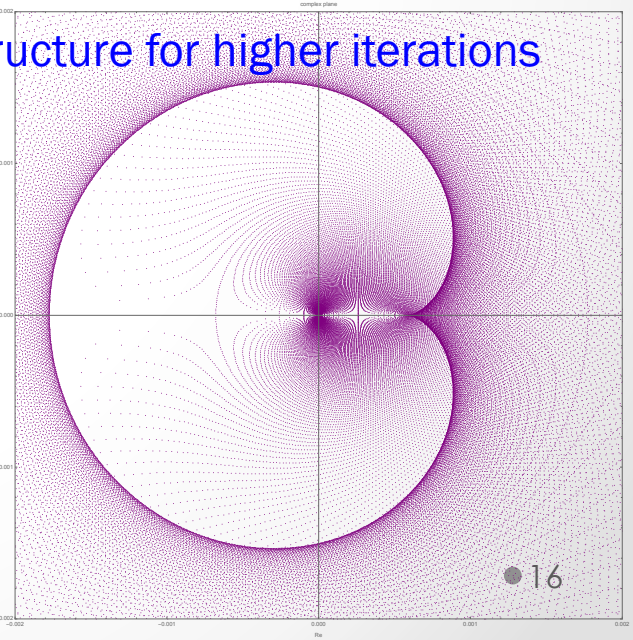
23 iterations



24 iterations



25 iterations



Richer near axis structure for higher iterations

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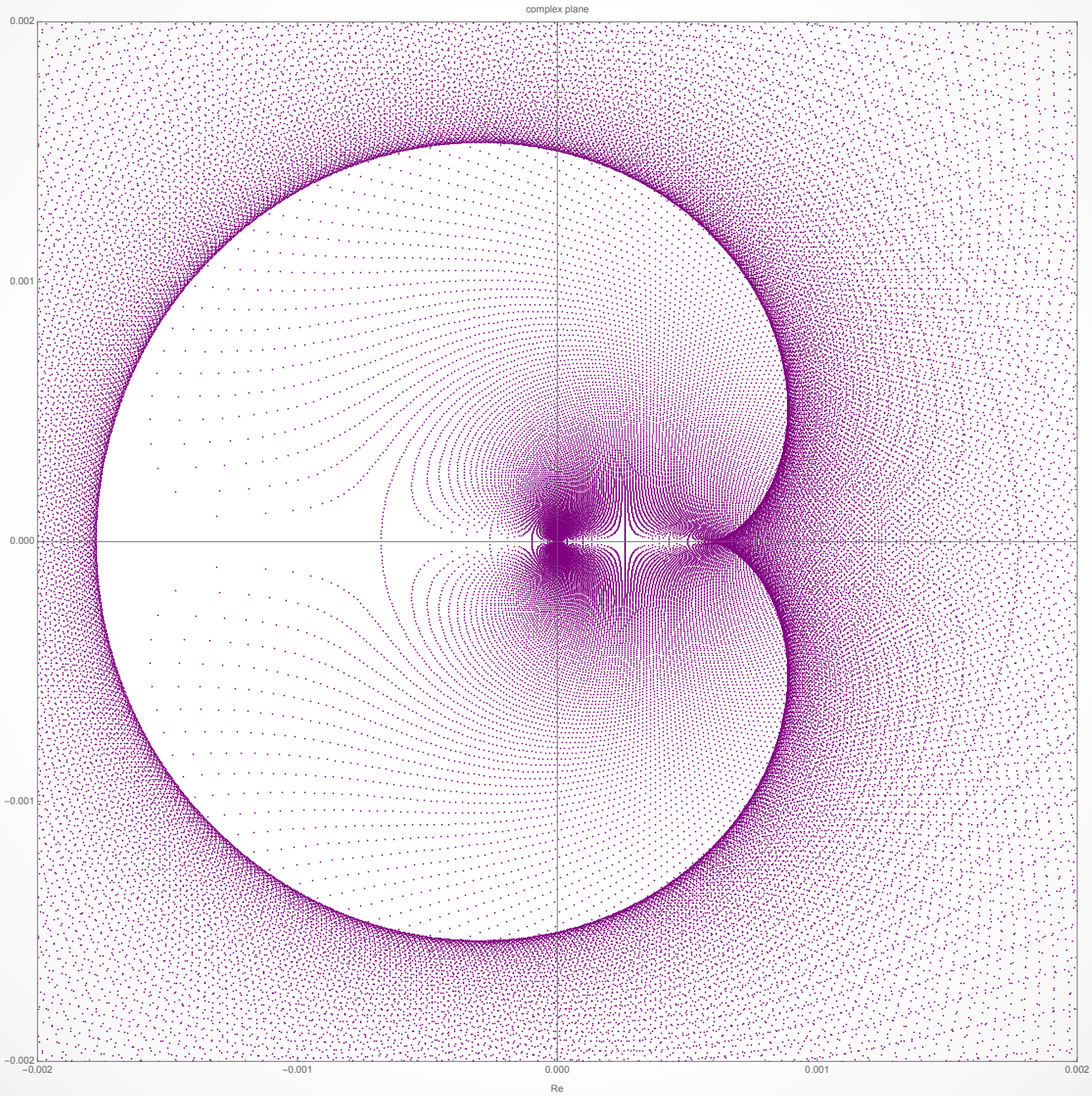
Zooming in at the center...

25 iterations



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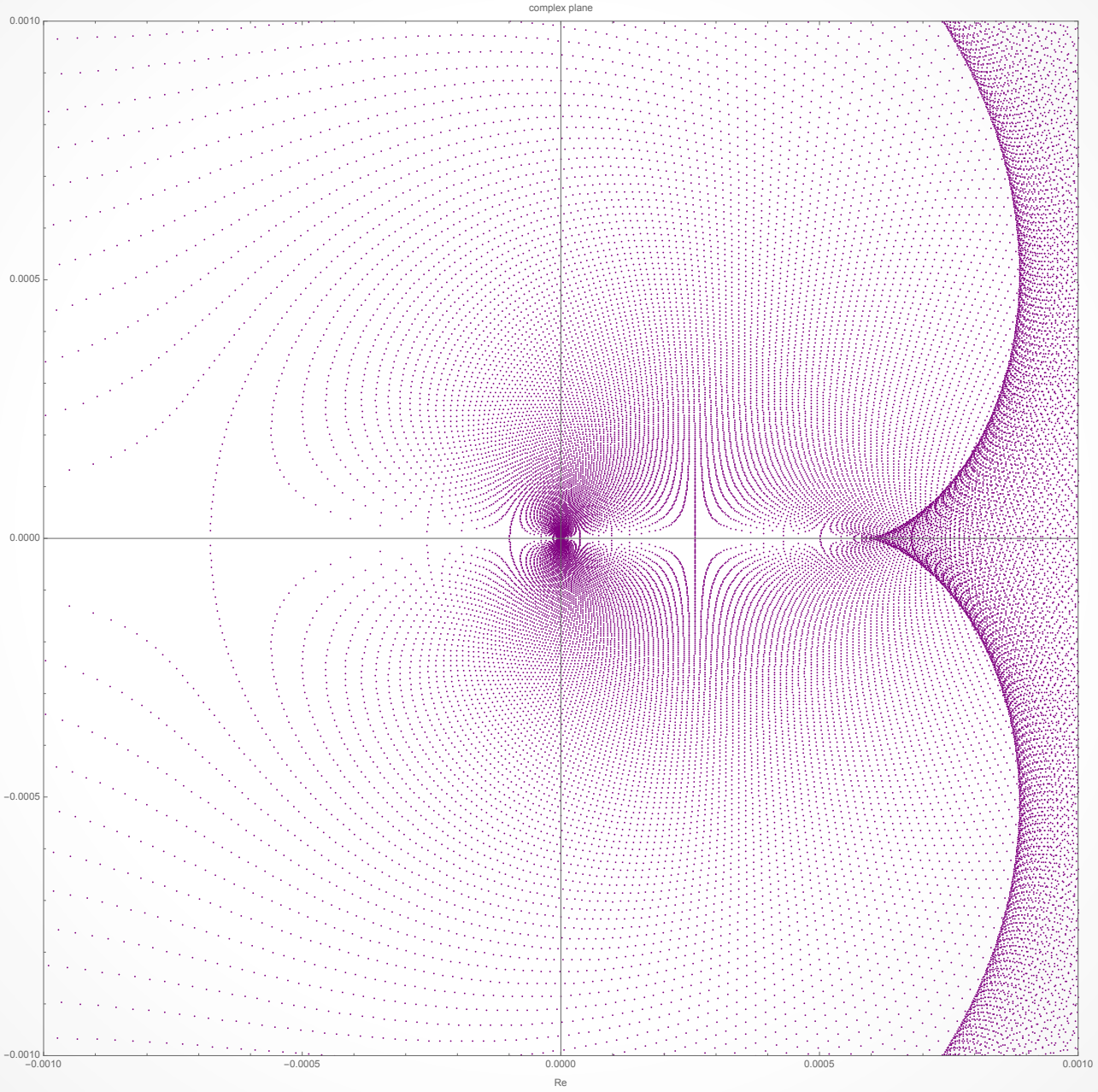
Zooming in at the center…

25 iterations



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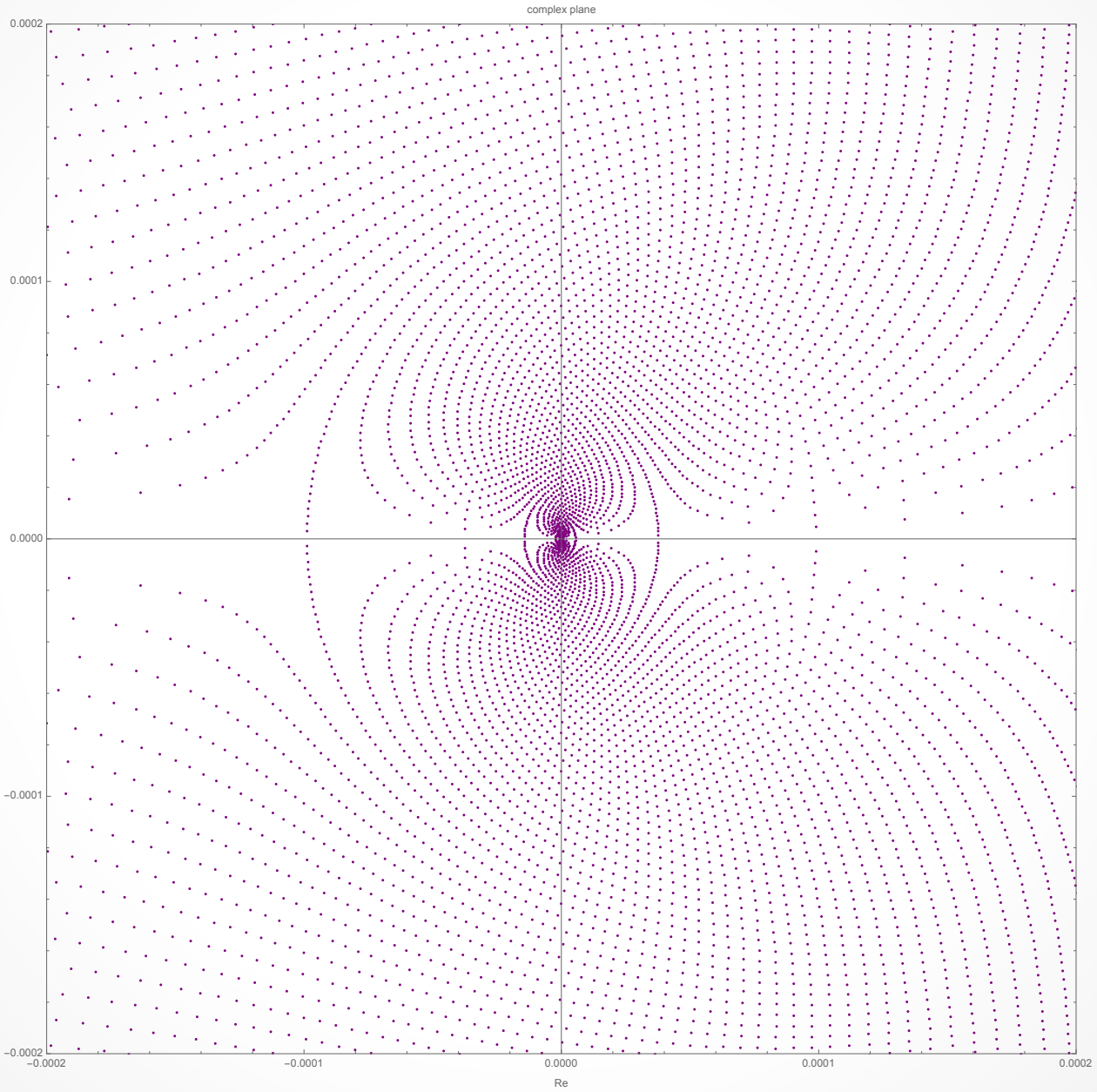
Zooming in at the center...

25 iterations



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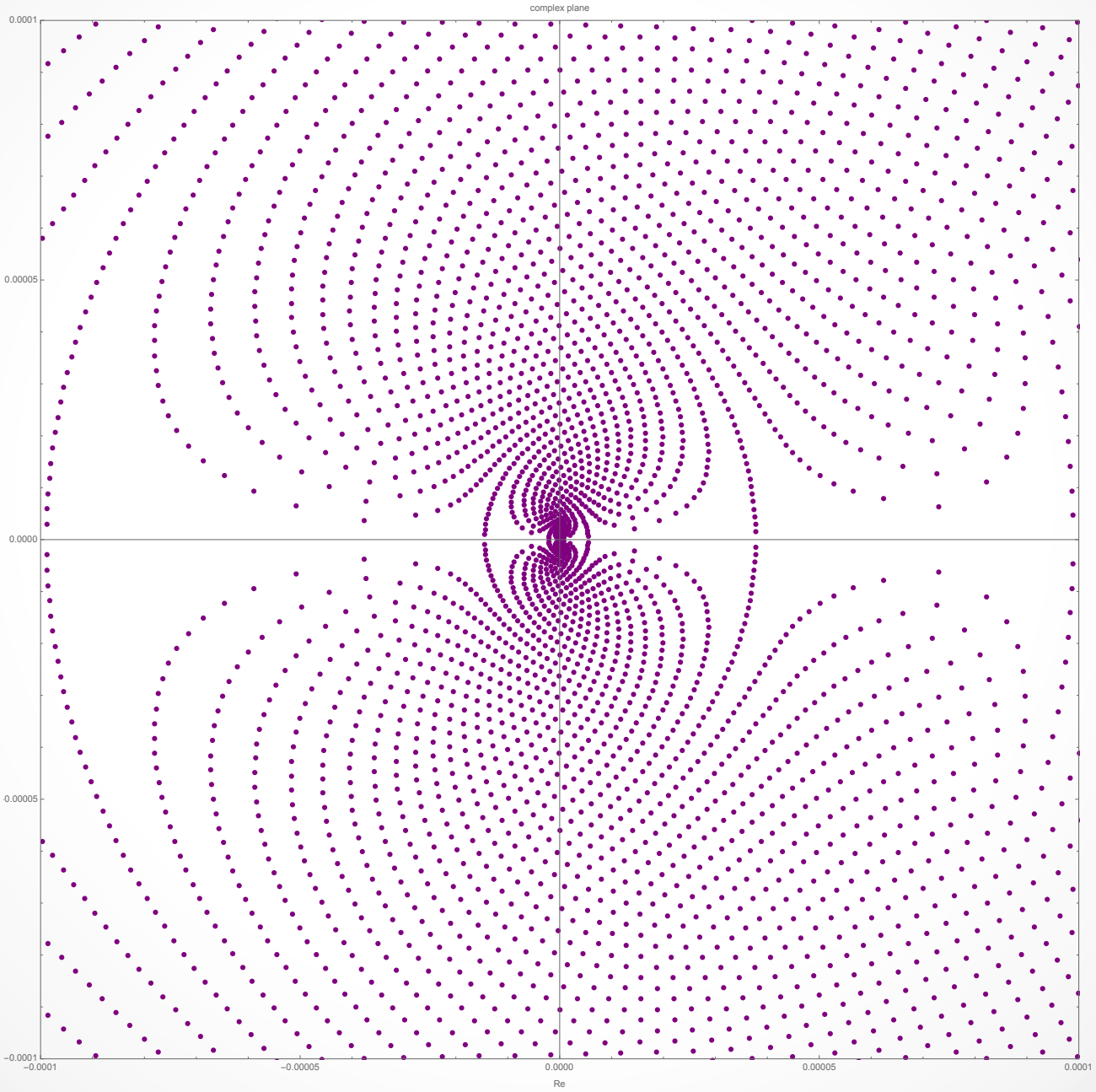
Zooming in at the center...

25 iterations



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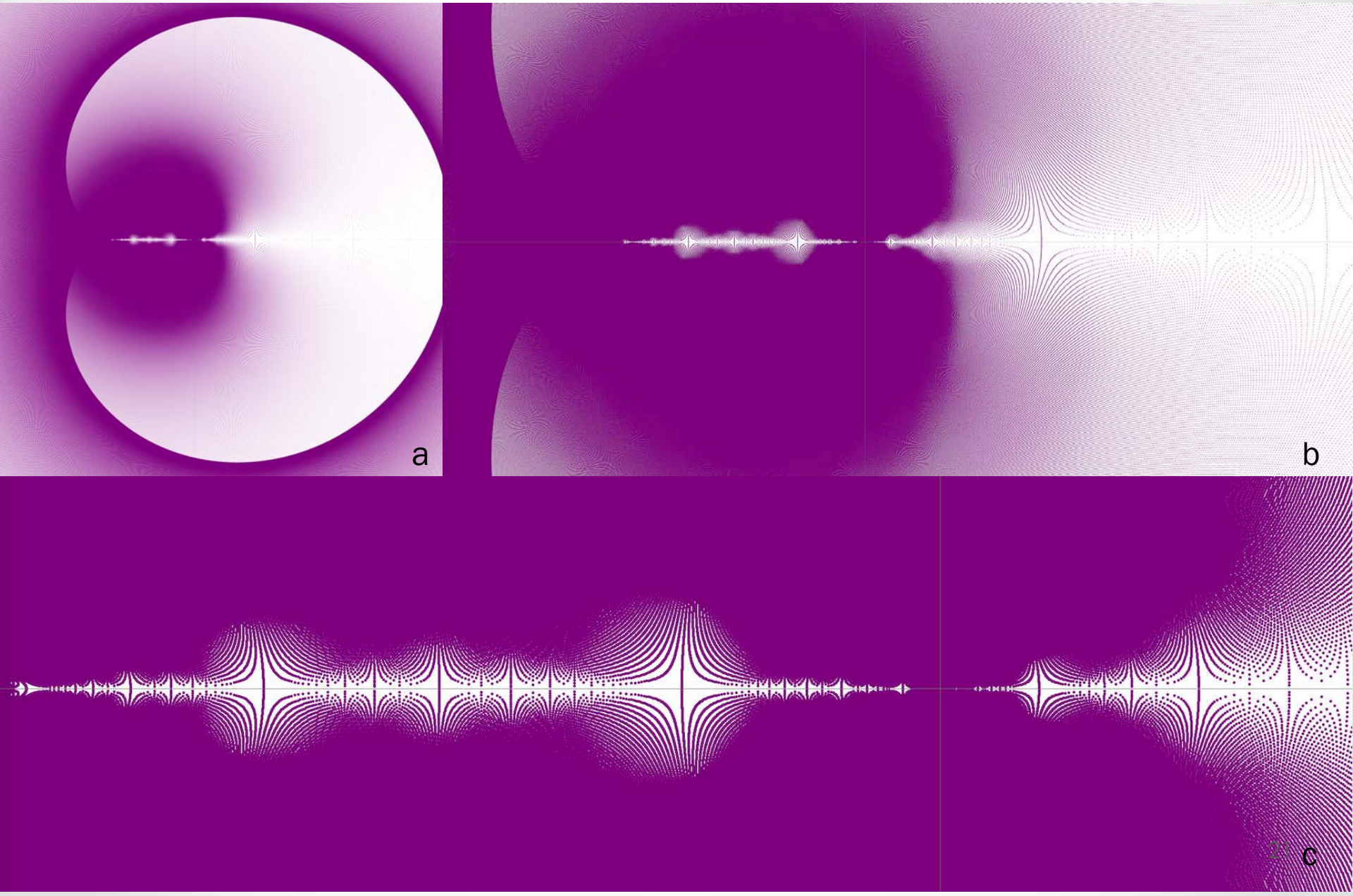
34 iterations – zooming near axis

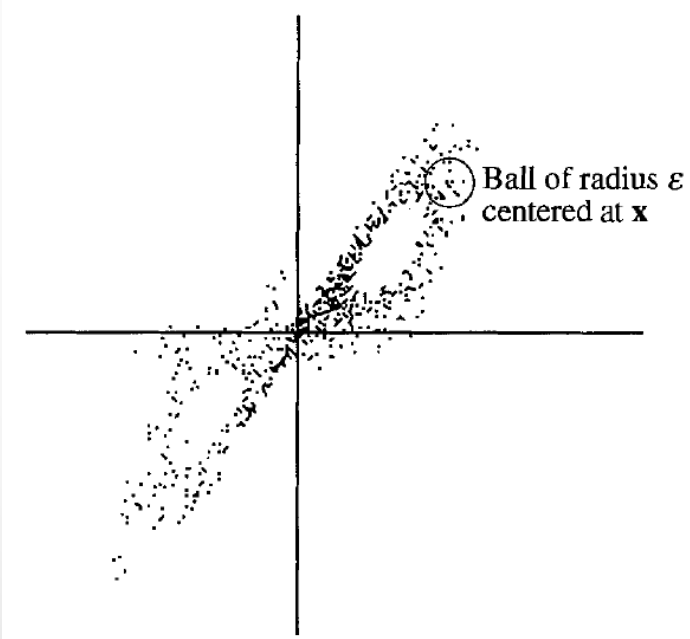


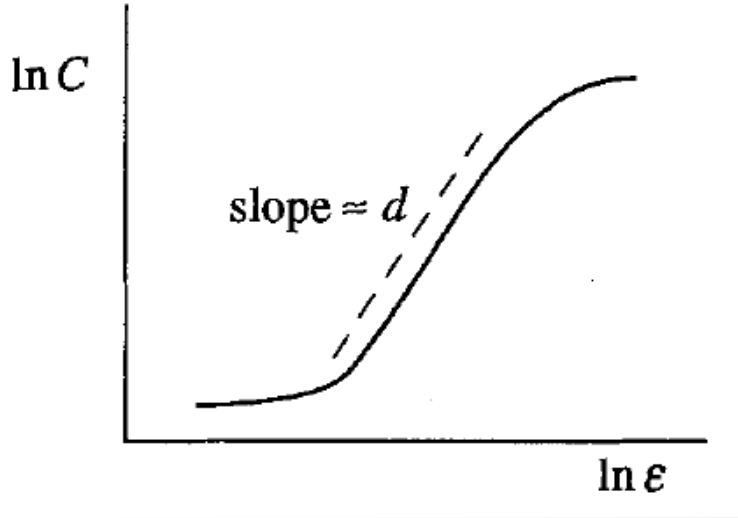
Pointwise dimension

Pointwise dimension, proposed by Grassberger and Pracacia, is a standardized way to calculate the fractal dimension for a chaotic system that settles down to a strange attractor in phase space, which is our case here.

In this method, $N_x(\varepsilon)$ denotes the number of point in the set that is inside of a ball of radius x.

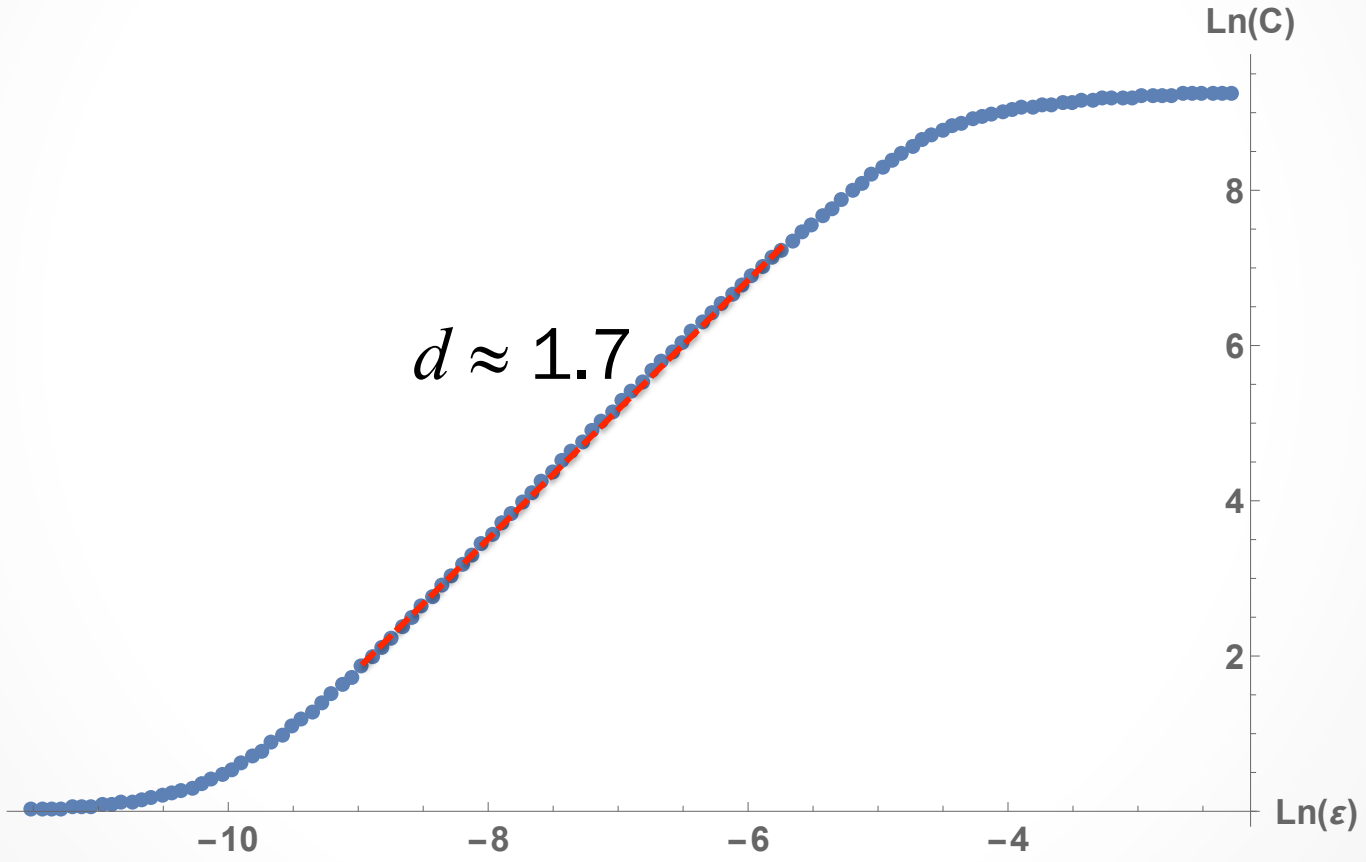
Typically, $N_x(\varepsilon) \propto \varepsilon^d$, where d is the **pointwise** dimension at x. For the overall dimension of the whole set, the average of $N_x(\varepsilon)$, denoted by $C(\varepsilon)$ is used.





Picture made by Scott Sutherland at Stony Brook Mathematics

Pointwise dimension of the cycloidal Fibonacci fractal



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The derivatives of the cycloidal Fibonacci fractal

- Truncation at the head
- Truncation at the tail
- Partial scramble of the Fibonacci order
- Modification of the L/S ratio

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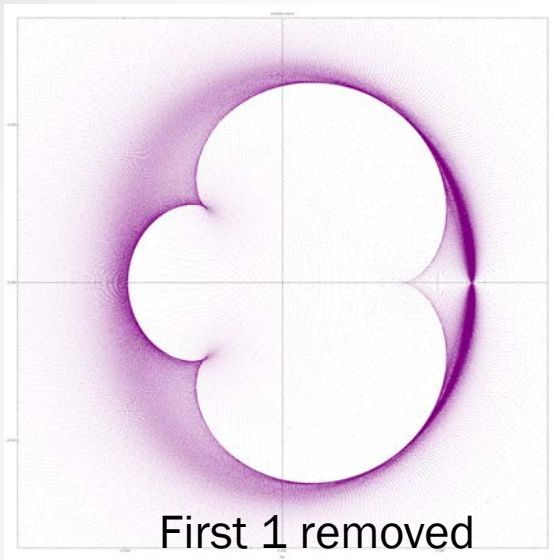
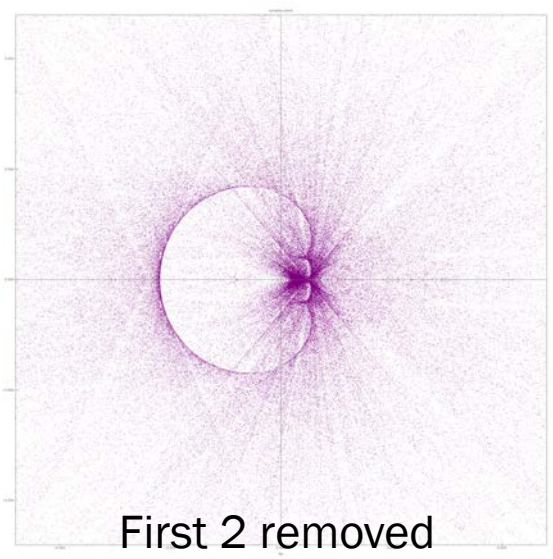
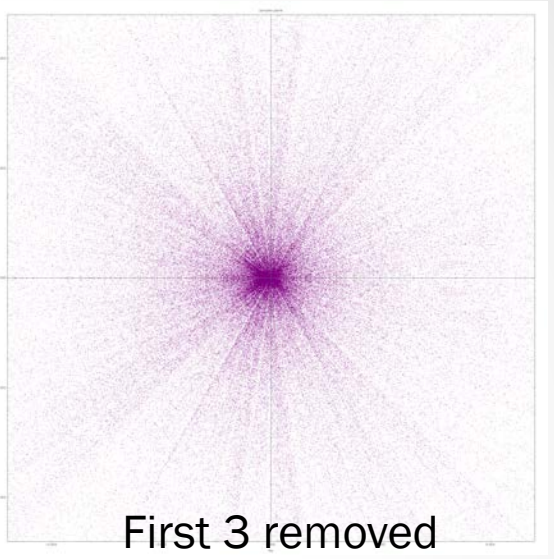
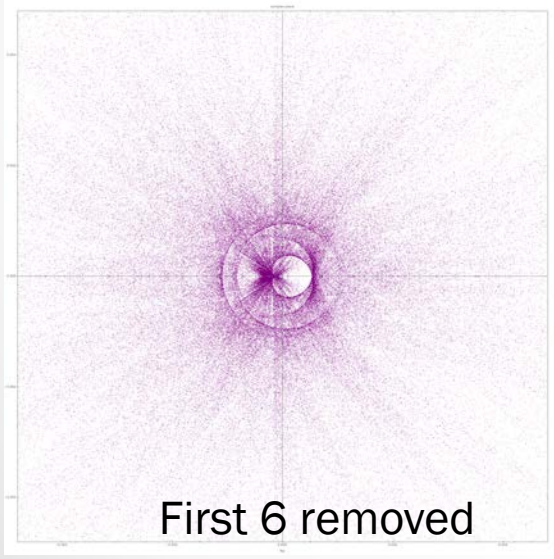
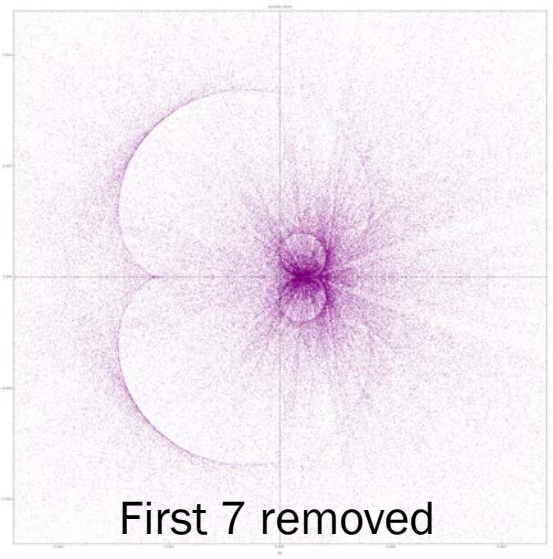
Truncation at the head

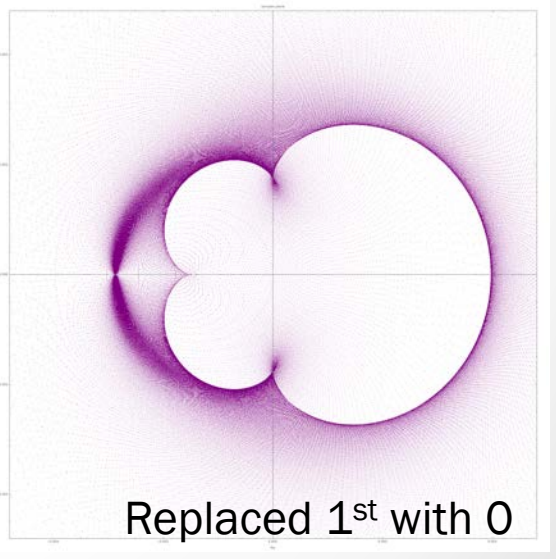

First 1 removed


First 2 removed

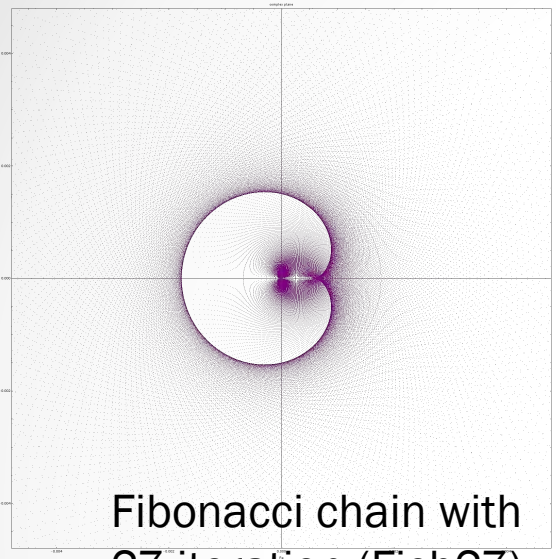

First 3 removed


First 6 removed

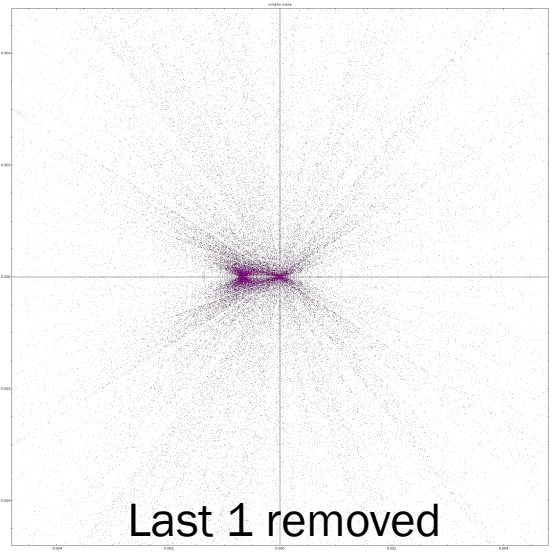

First 7 removed


Replaced 1st with 0

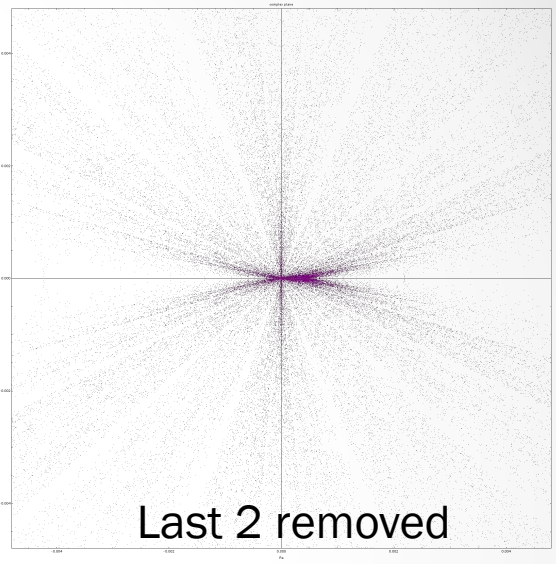
Truncation at the tail



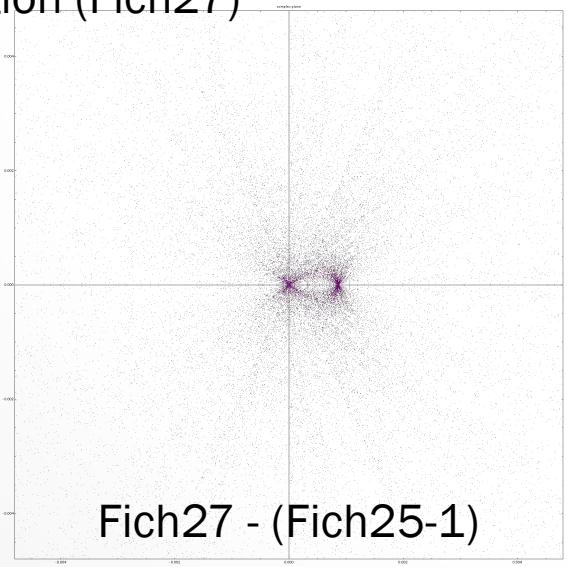
Fibonacci chain with 27 iteration (Fich27)



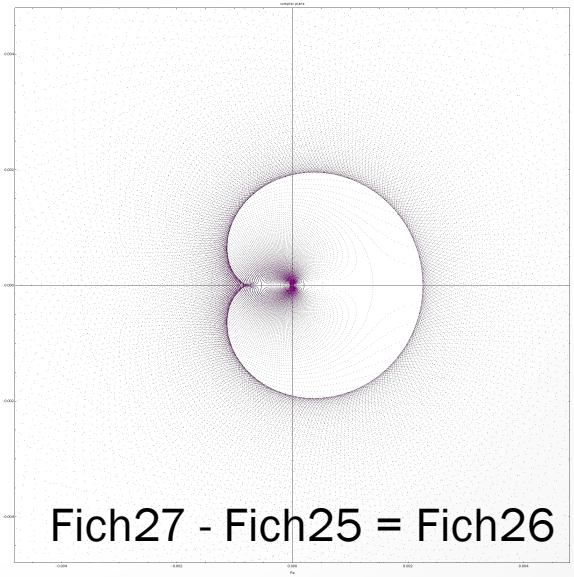
Last 1 removed



Last 2 removed



Fich27 - (Fich25-1)



Fich27 - Fich25 = Fich26

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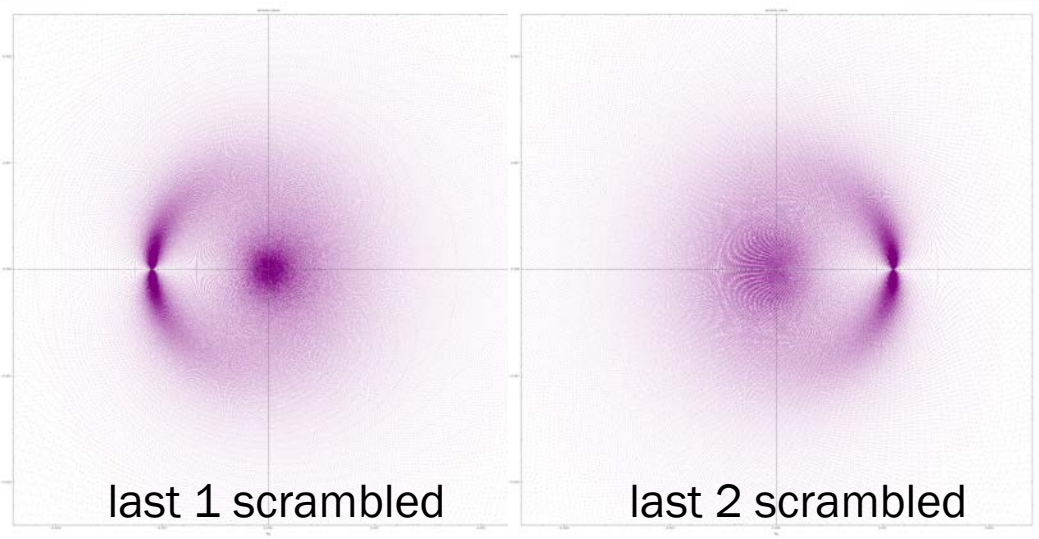
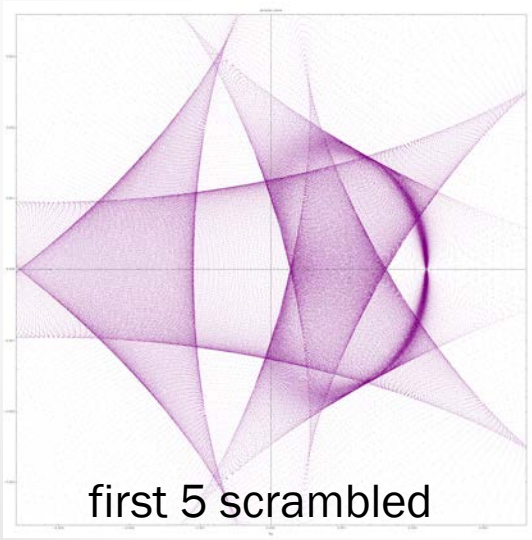
Partial scramble of the Fibonacci order

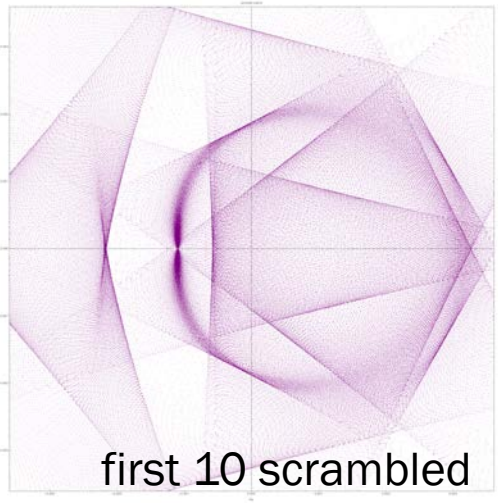


last 1 scrambled

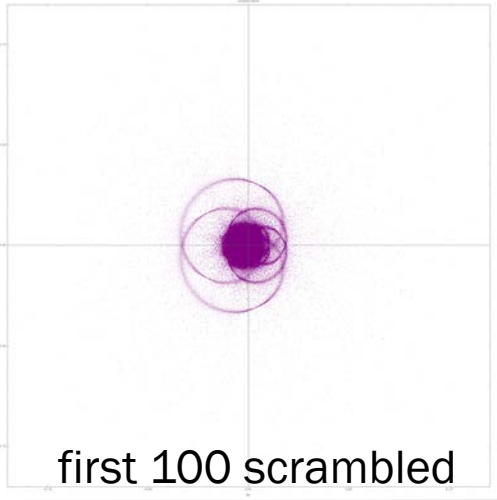
last 2 scrambled



first 5 scrambled

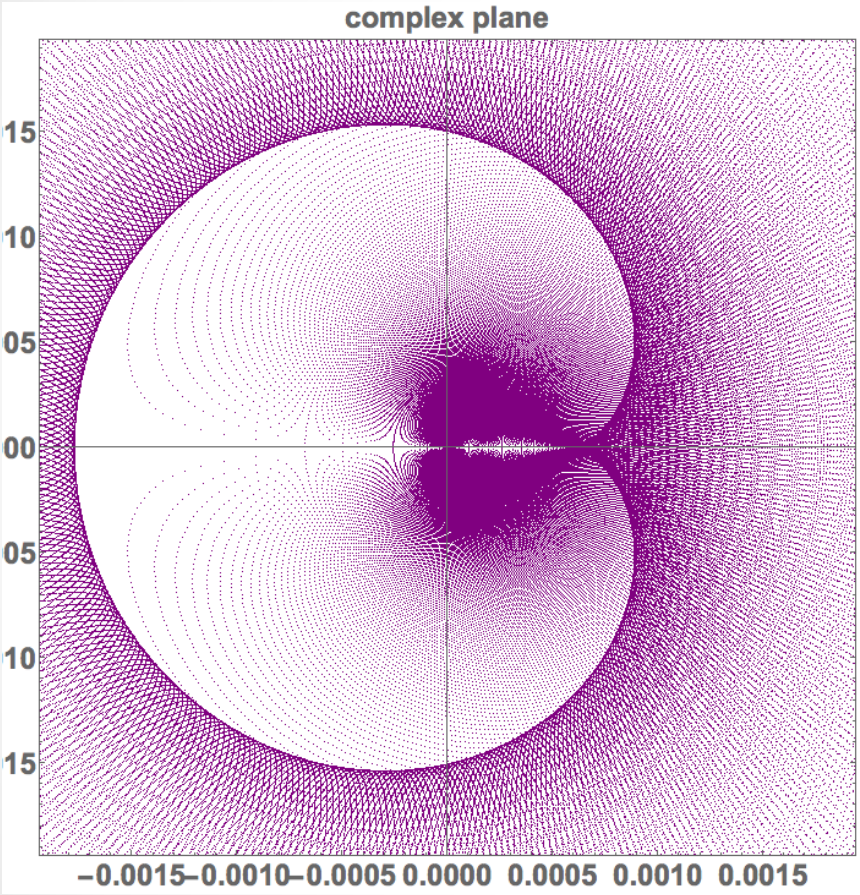


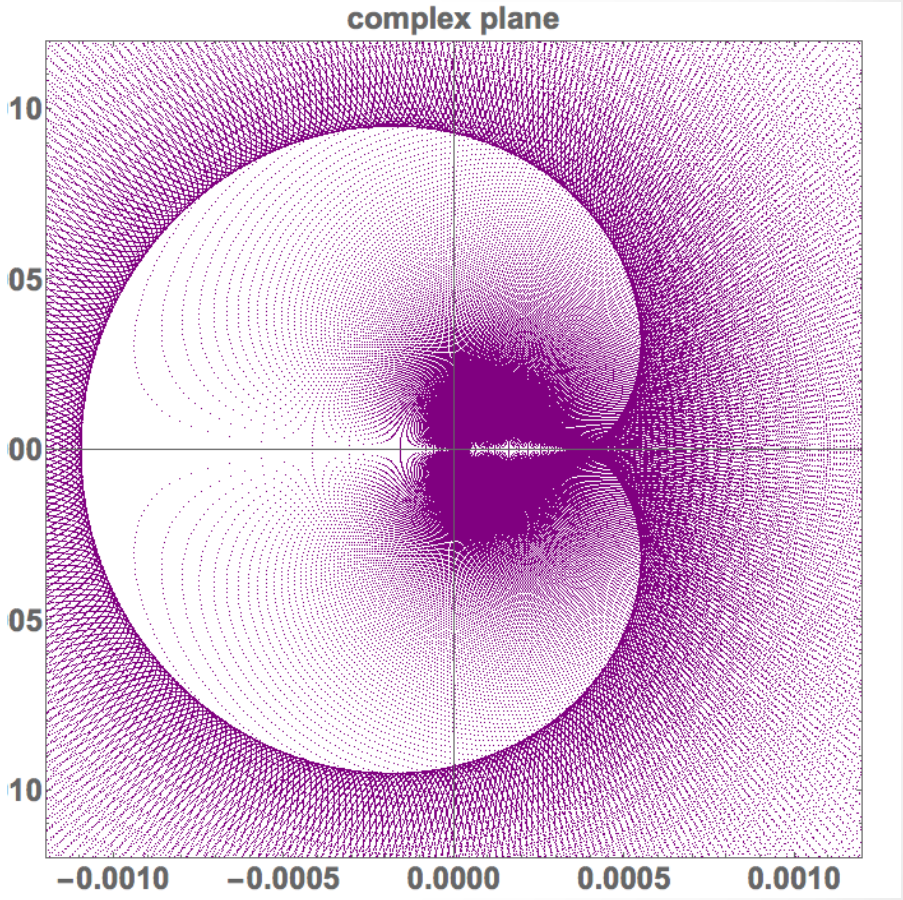
first 10 scrambled



first 100 scrambled

Modification of the L/S ratio



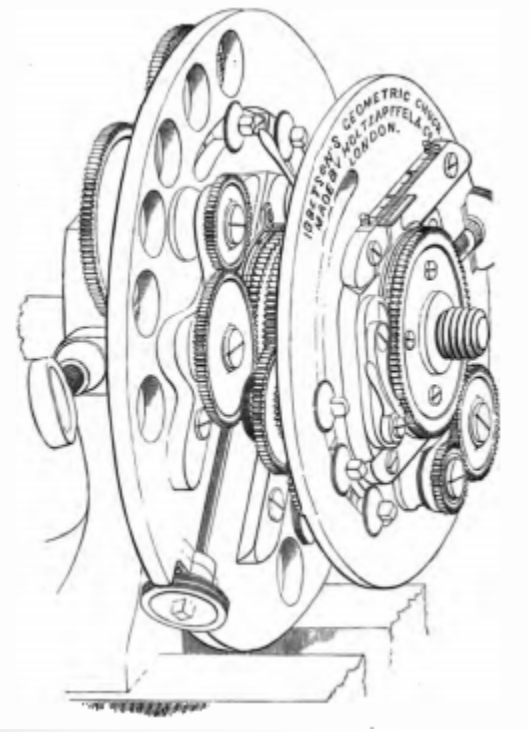


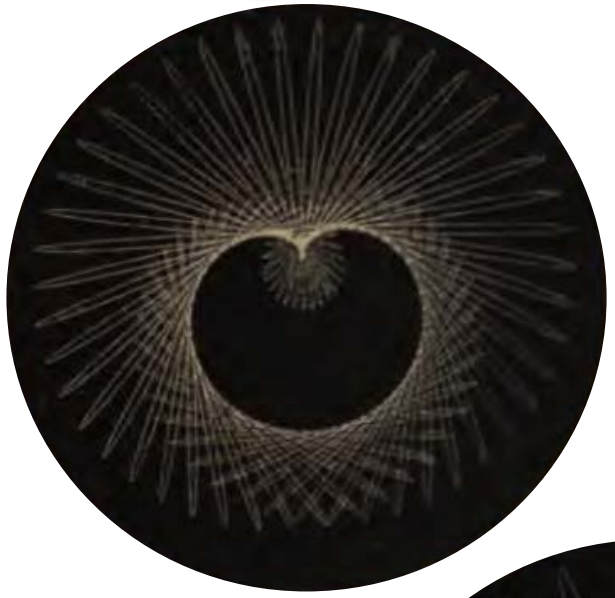
S=1 and L=ϕ

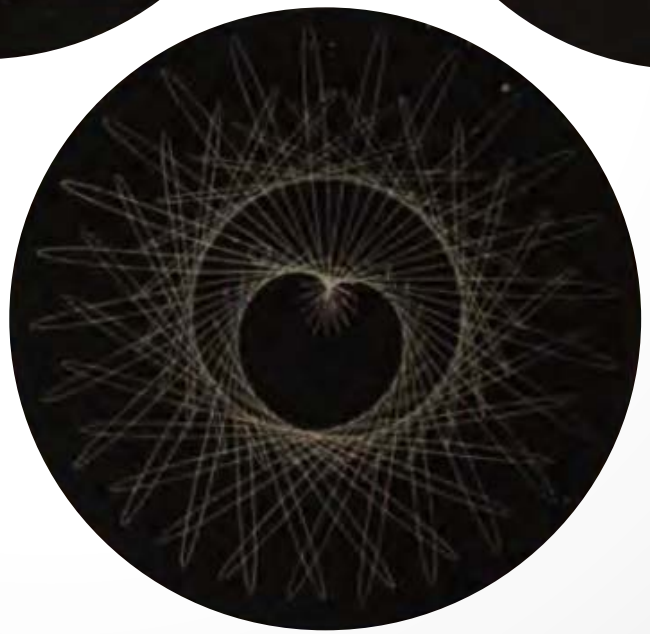
S=1 and L=2

Pattern identical but scaled by a factor of ϕ

Comparison with the cycloids from the geometric chuck







Picture taken from the book *index to geometric chuck* by Thomas Sebastian Bazley, M.A. 1875

QUANTUM
GRAVITY RESEARCH

Summary

- Cycloidal fractal signature is observed in the Fourier space of the Fibonacci chain. The Pointwise dimension is calculated to be about 1.7.
- The cardioid in the fractal pattern scales down by Φ/2 as the iteration goes up by 1. Also its orientation undergoes mirror flip between consecutive iterations of the Fibonacci chain.
- The derivatives of the Fibonacci fractal show that the cycloidal fractal pattern is sensitive to the changes in the Fibonacci order, but not to the L/S ratio.
- Also the cycloidal pattern suggests a connection between the Fibonacci fractal and the cyloids created by the geometric chuck.