The Quasicrystalline Spin-Network — A Chiral Icosahedral Quasicrystal Derived from E_8

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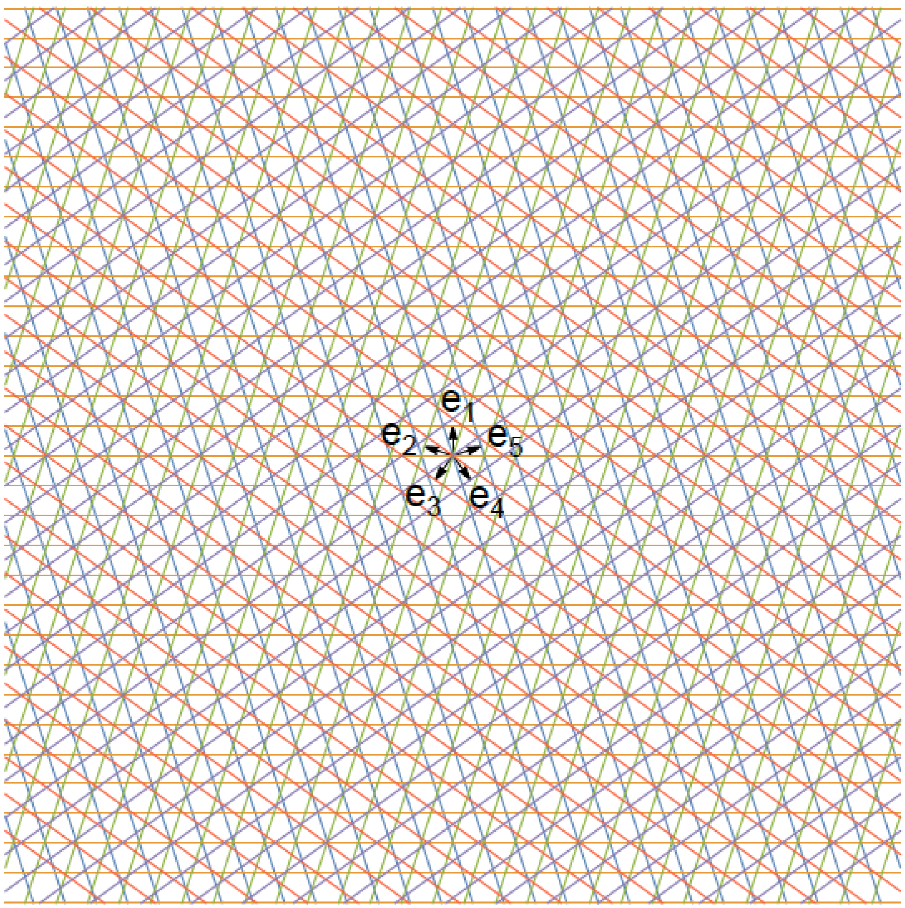
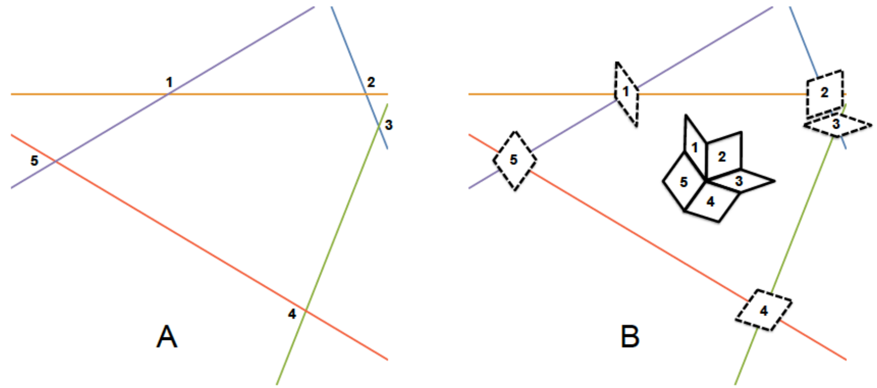
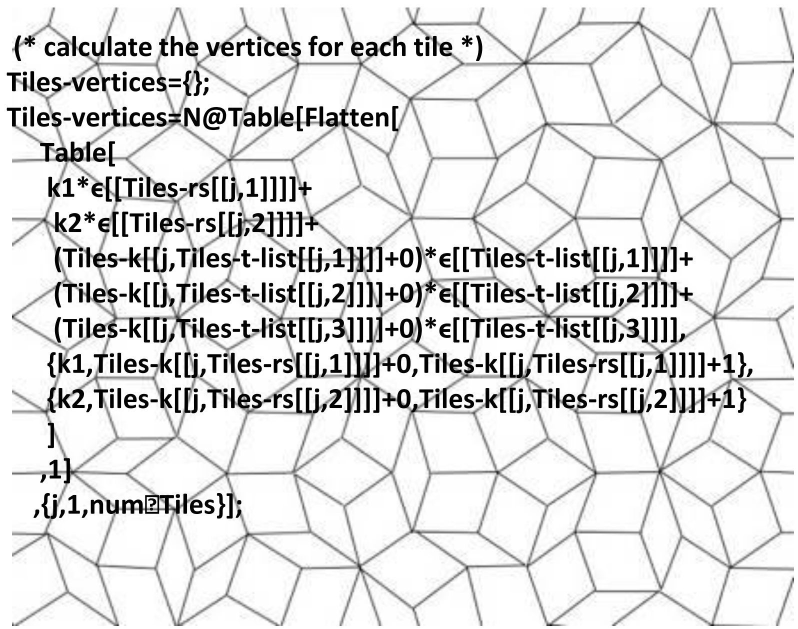
We present the construction of an icosahedral quasicrystal, a quasicrystalline spin network, obtained by spacing the parallel planes in an icosagrid according to the Fibonacci sequence. This quasicrystal can be thought of as a golden composition of five sets of Fibonacci tetragrids. This quasicrystal is made up of embedded quasicrystals that are *golden* compositions of the three-dimensional tetrahedral cross-sections of the Elser-Sloane quasicrystal, which is a four-dimensional cut-and-project set of the E_8 lattice. These compound quasicrystals are subsets of the quasicrystalline spin network, and the former can be enriched to form the latter. This creates a mapping between the quasicrystalline spin network and the E_8 lattice.

Method I:FIBONACCI MULTIGRID METHOD



Pentagrid





```
(* calculate the vertices for each tile *)
Tiles-vertices={};
Tiles-vertices=N@Table[Flatten[
   Table[
    k1*ϵ[[Tiles-rs[[j,1]]]]+
     k2*ϵ[[Tiles-rs[[j,2]]]]+
     (Tiles-k[[j,Tiles-t-list[[j,1]]]]+0)*ϵ[[Tiles-t-list[[j,1]]]]+
     (Tiles-k[[j,Tiles-t-list[[j,2]]]]+0)*ϵ[[Tiles-t-list[[j,2]]]]+
     (Tiles-k[[j,Tiles-t-list[[j,3]]]]+0)*ϵ[[Tiles-t-list[[j,3]]]],
    {k1,Tiles-k[[j,Tiles-rs[[j,1]]]]+0,Tiles-k[[j,Tiles-rs[[j,1]]]]+1},
    {k2,Tiles-k[[j,Tiles-rs[[j,2]]]]+0,Tiles-k[[j,Tiles-rs[[j,2]]]]+1}
    ]
   ,1]
  ,{j,1,num□Tiles}];
```

```
 (* calculate the vertices for each tile *)
Tiles□vertices={};
Tiles□vertices=N@Table[Flatten[
    Table[
     k1*ϵ[[Tiles□rs[[j,1]]]]+
      k2*ϵ[[Tiles□rs[[j,2]]]]+
      (Tiles□k[[j,Tiles□t□list[[j,1]]]]+0)*ϵ[[Tiles□t□list[[j,1]]]]+
      (Tiles□k[[j,Tiles□t□list[[j,2]]]]+0)*ϵ[[Tiles□t□list[[j,2]]]]+
      (Tiles□k[[j,Tiles□t□list[[j,3]]]]+0)*ϵ[[Tiles□t□list[[j,3]]]],
     {k1,Tiles□k[[j,Tiles□rs[[j,1]]]]+0,Tiles□k[[j,Tiles□rs[[j,1]]]]+1},
     {k2,Tiles□k[[j,Tiles□rs[[j,2]]]]+0,Tiles□k[[j,Tiles□rs[[j,2]]]]+1}
     ]
    ,1]
   ,{j,1,num□Tiles}];
```

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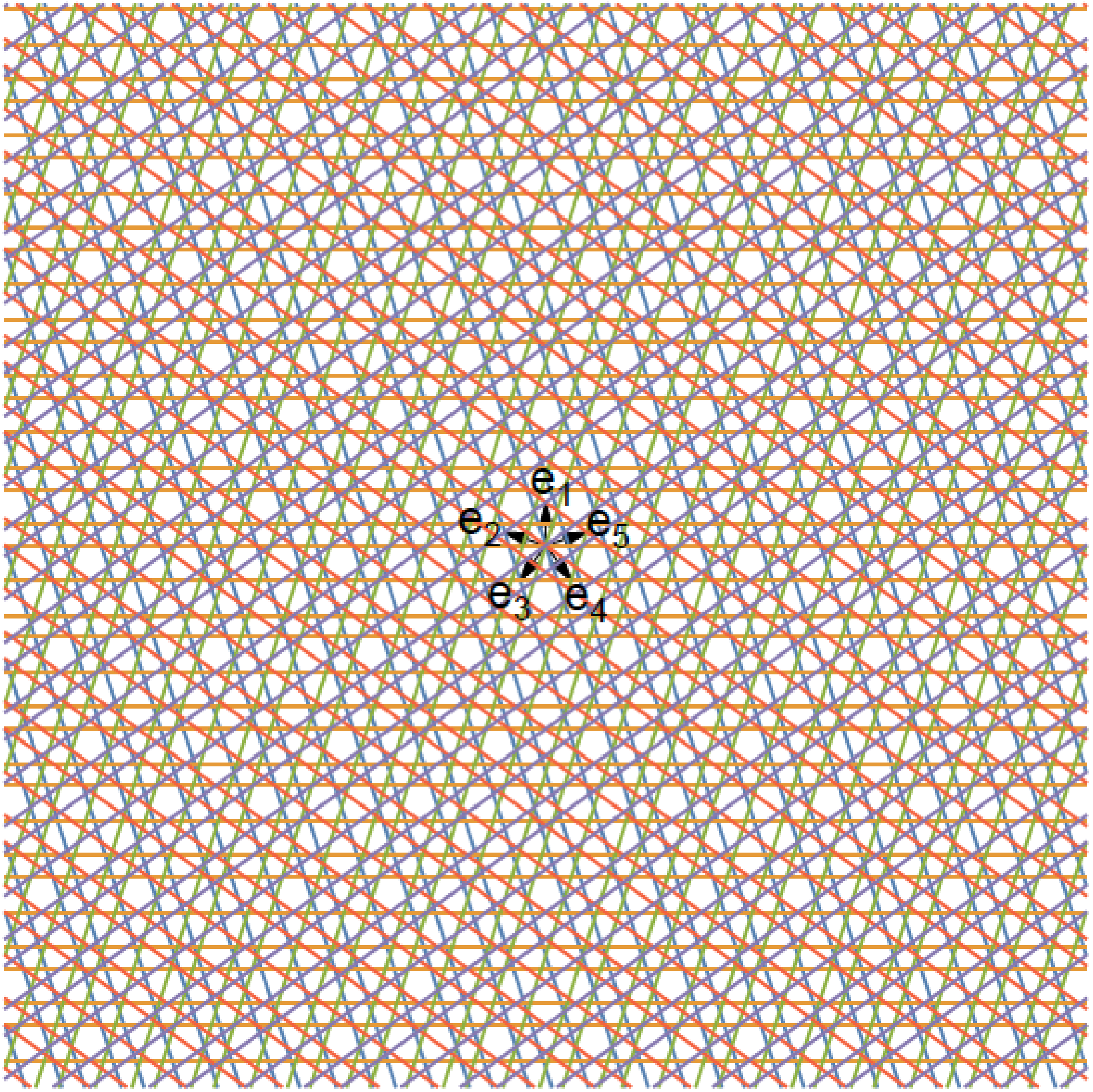

Fibonacci spaced Pentagrid

Method I:FIBONACCI MULTIGRID METHOD

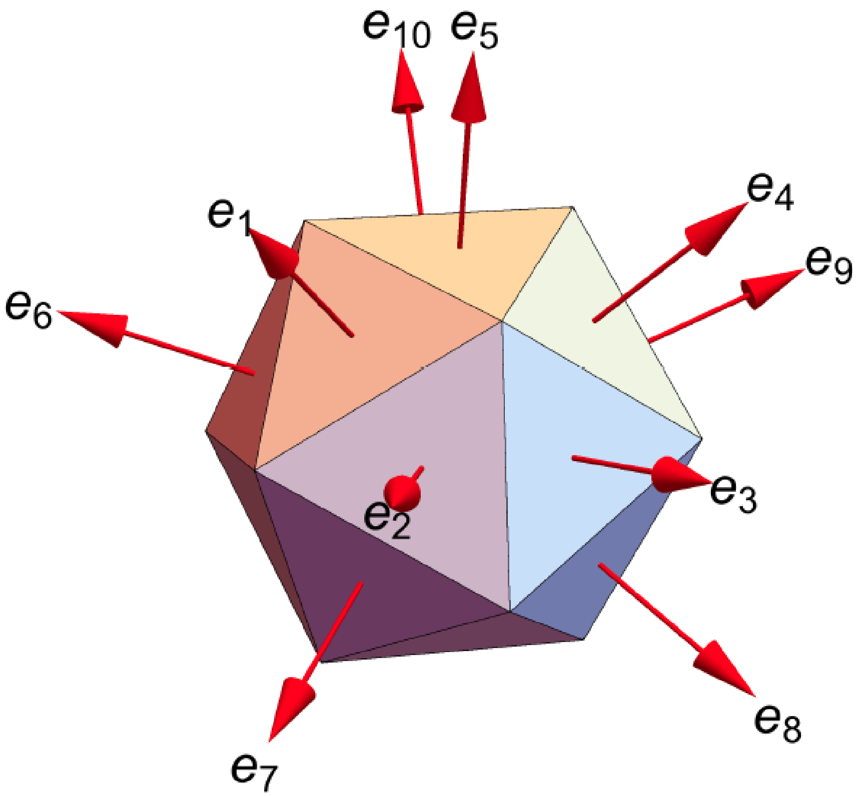

Figure 4: *The norm vectors of the icosagrid:* e_1, e_2,...,e_{10}.

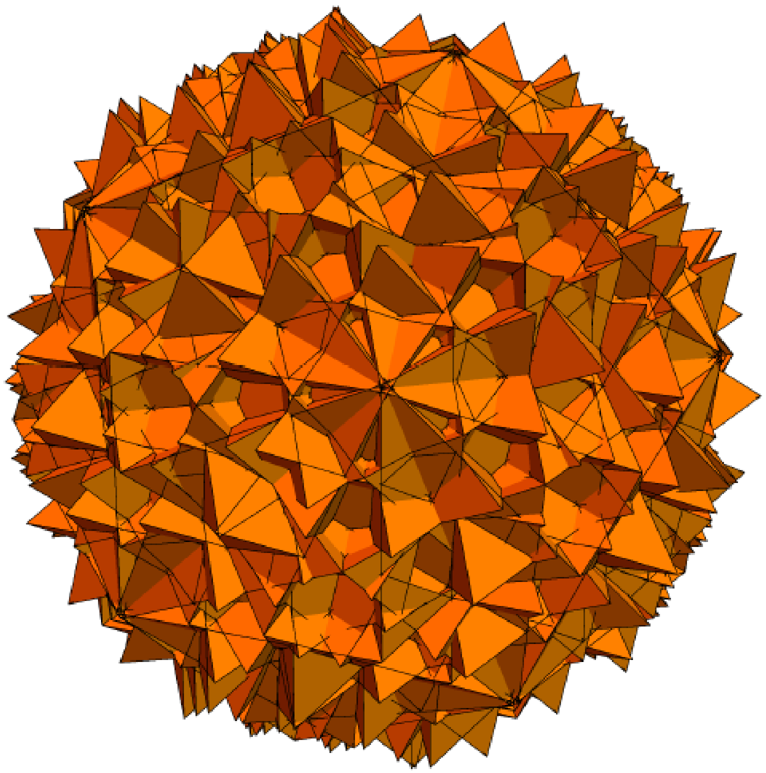

Figure 5: *Icosagrid with regular tetrahedral cells shown.*

Method I:FIBONACCI MULTIGRID METHOD

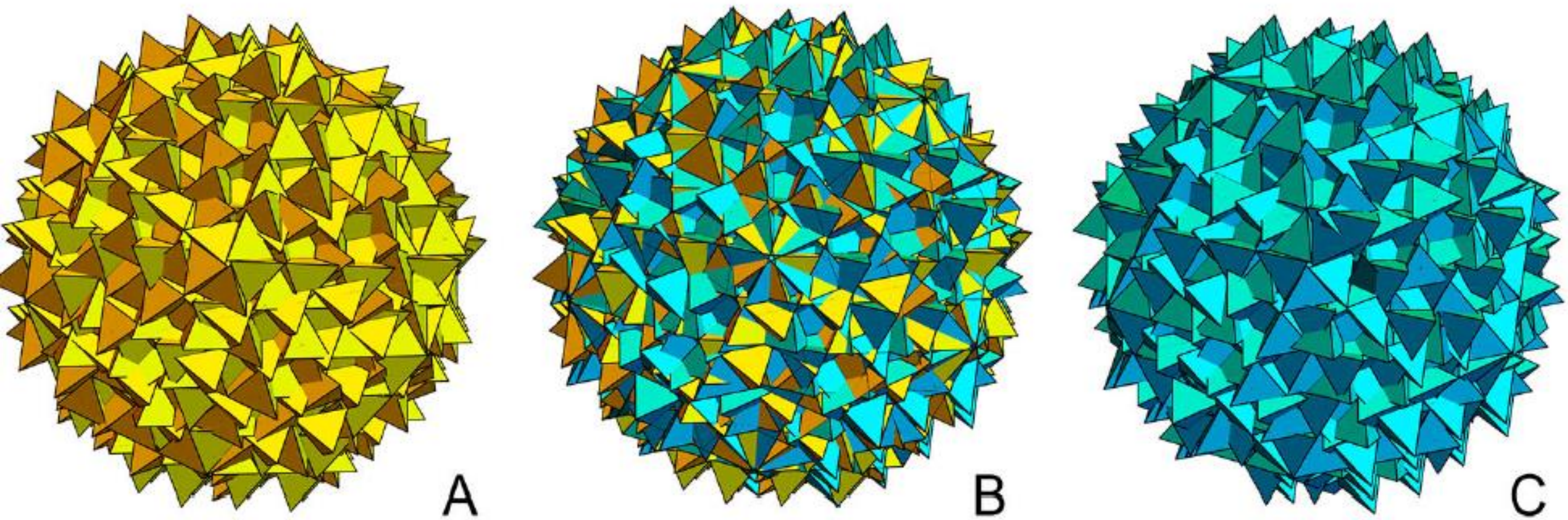

Figure 6: *Icosagrid (B) separated into two opposite chiralities: left A) and right C).*

Method I:FIBONACCI MULTIGRID METHOD

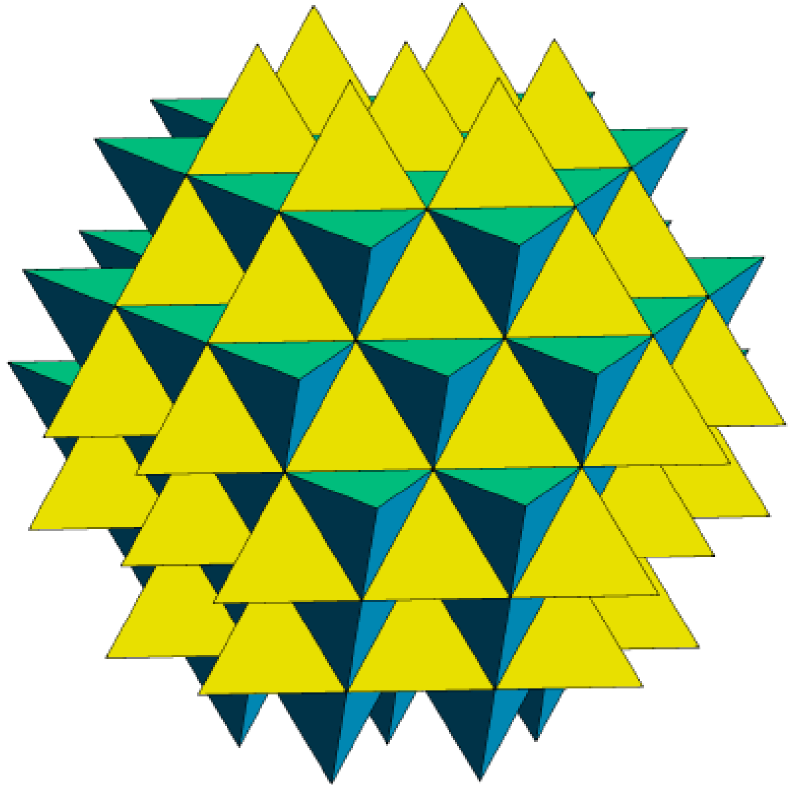

Figure 7: *Tetragrid with tetrahedral cells of two different orientations (yellow and cyan) and octahedral gaps.*

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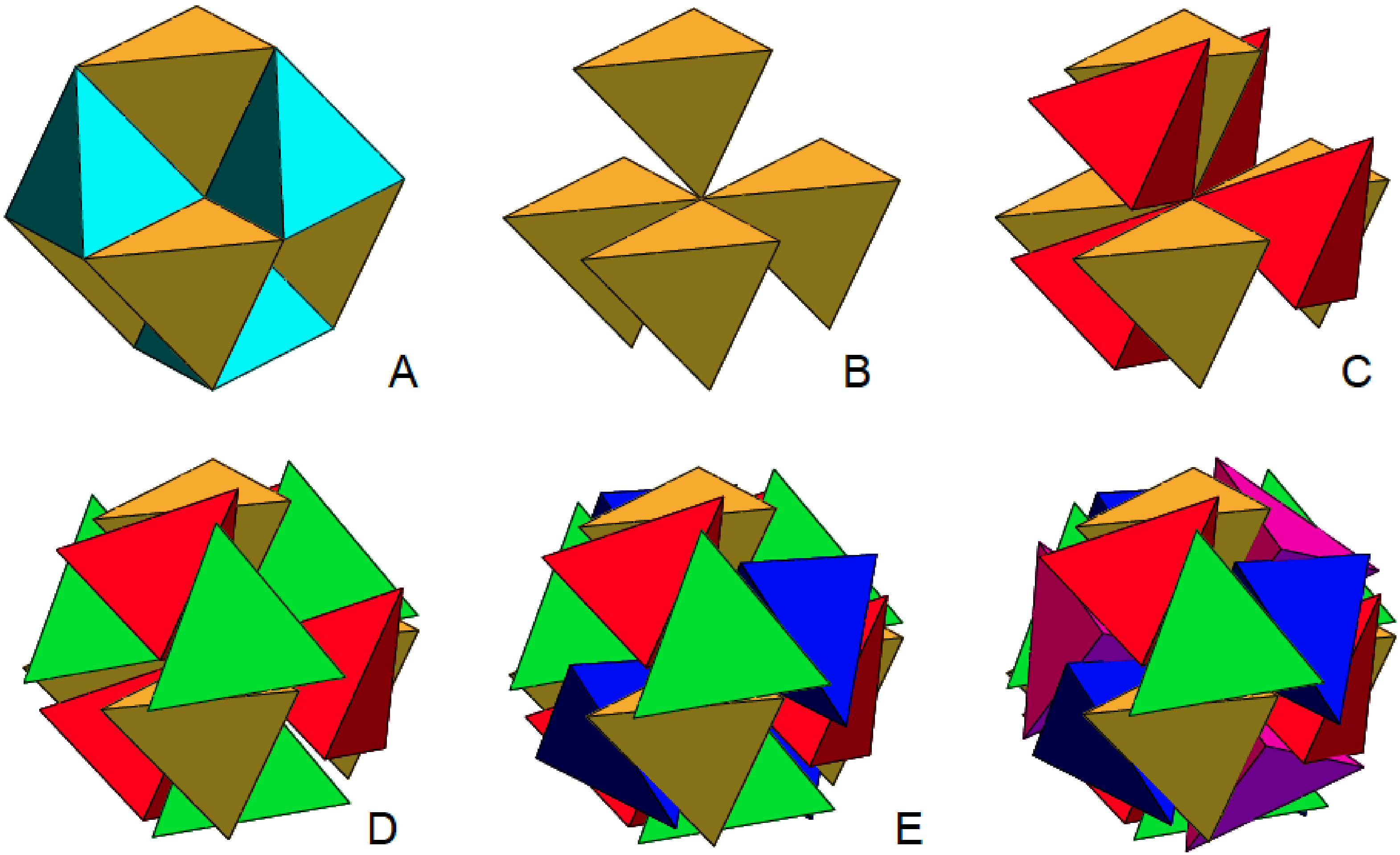

Figure 8: *(A) A small tetragrid local cluster with eight tetrahedral cells, four "up" and four "down". (B–F) The golden composition process).*

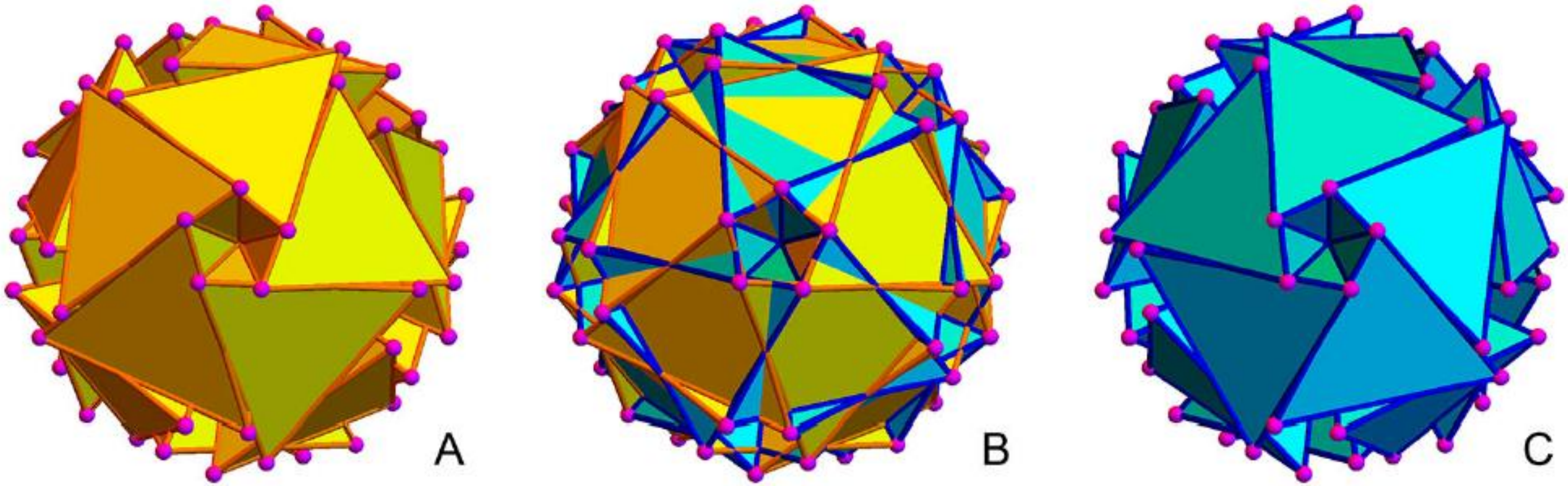

Figure 9: *(A) The right-twisted 20G, (B) the superposition of the left-twisted and right-twisted 20G, and (C) the left-twisted 20G.*

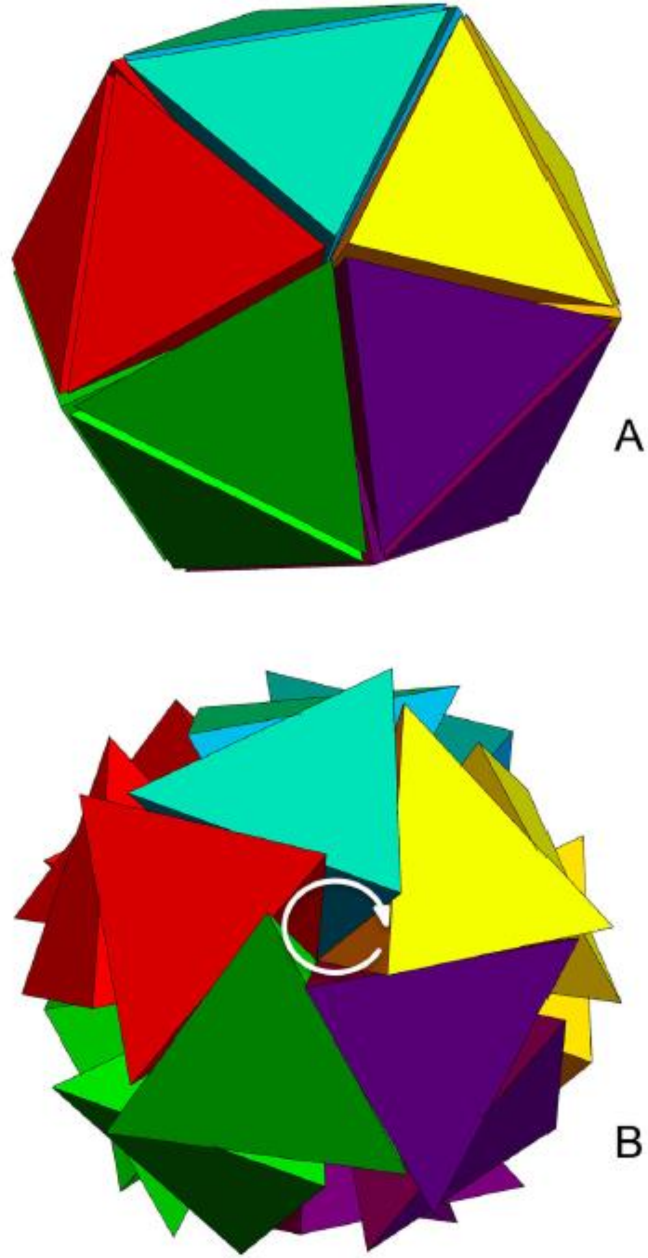

Figure 10: *(A) An evenly distributed vertex-sharing-20-tetrahedra cluster and (B) a twisted 20G with maximum plane class reduction.*

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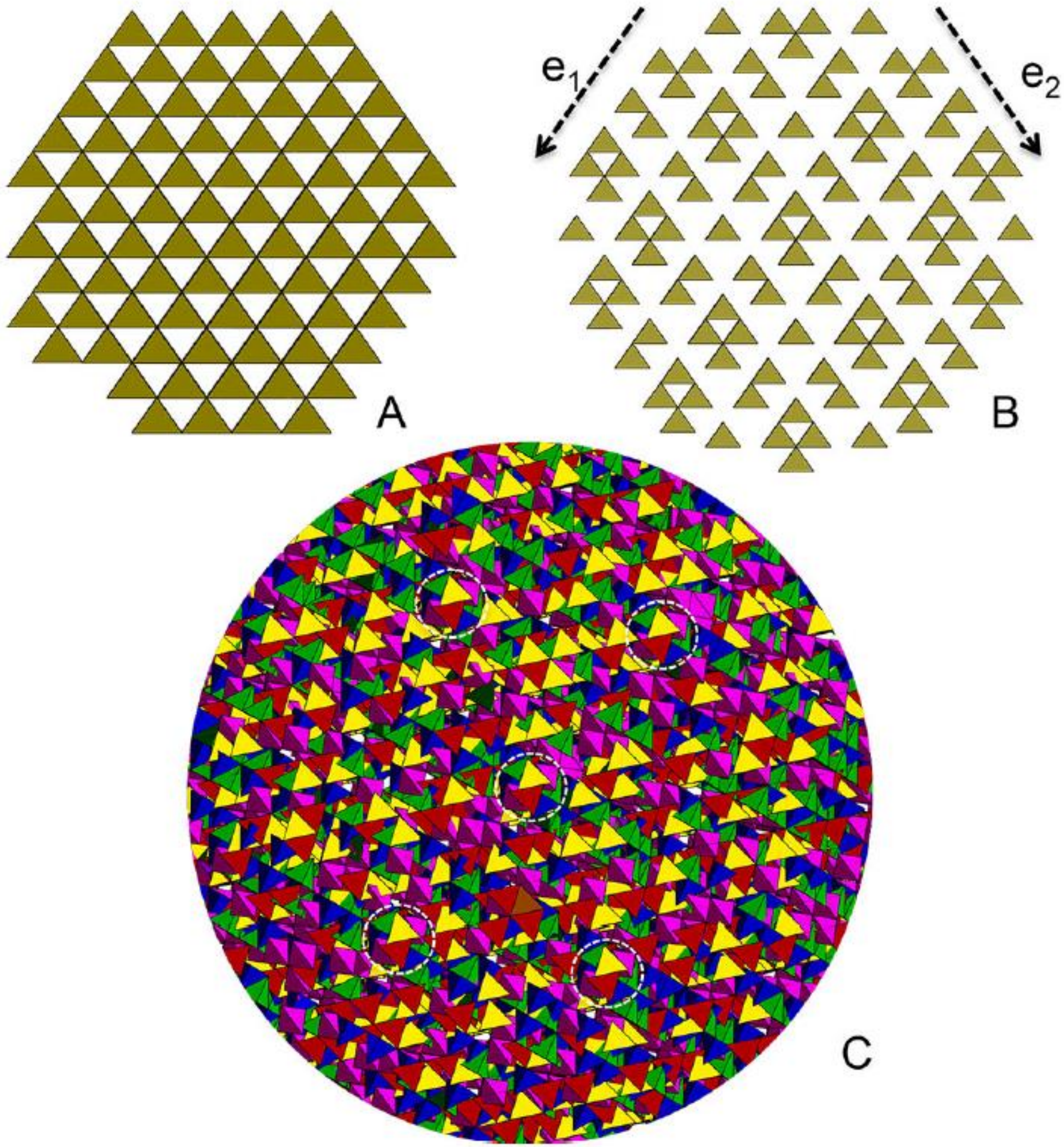


Figure 11: *(A) A 2D projection of a tetragrid, (B) a 2D projection of a Fibonacci tetragrid, and (C) a sample Fibonacci icosagrid. Notice that 20Gs formed up at the locations marked with white dotted circles.*

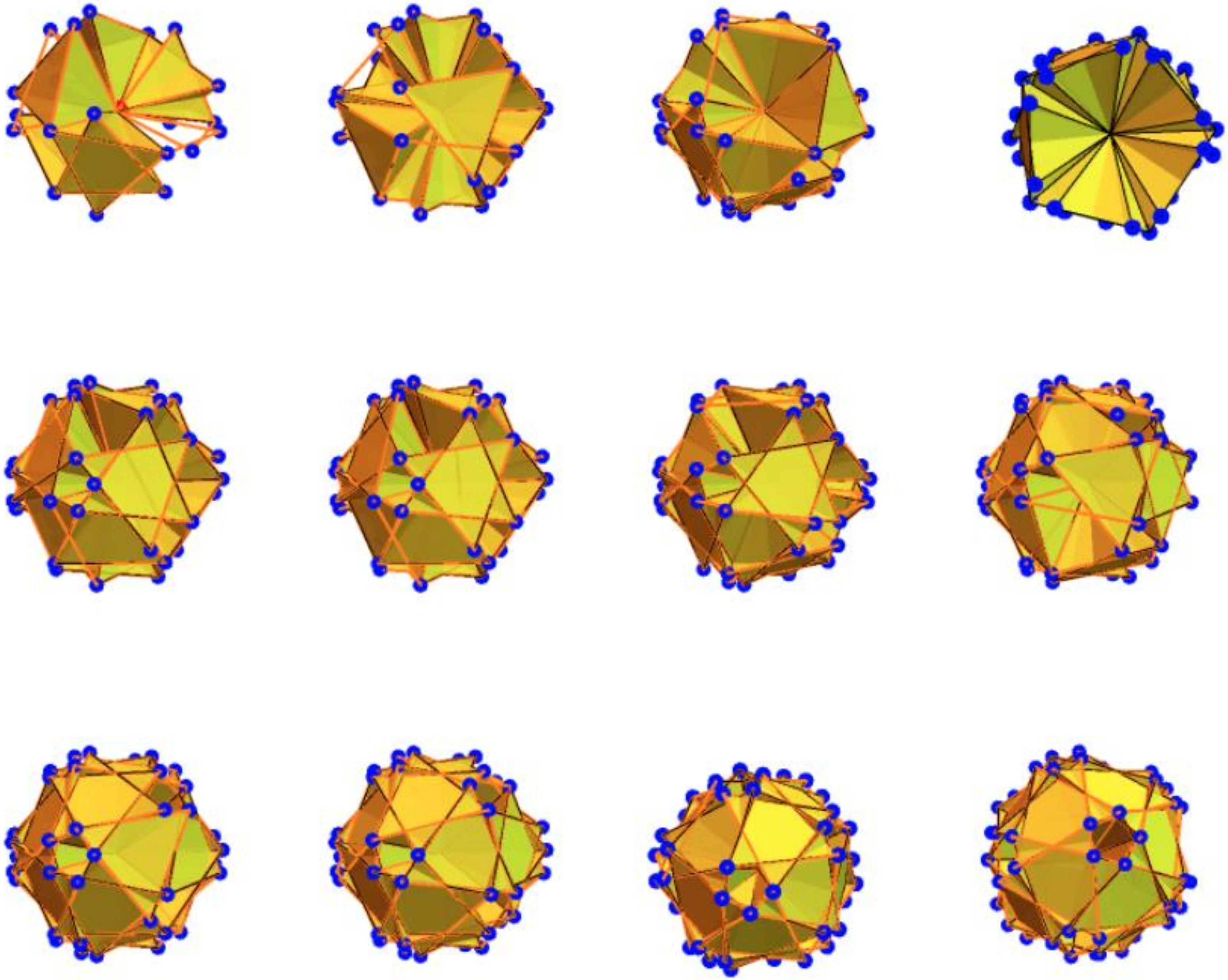


Figure 12: *A sampling of vertex configurations in the Fibonacci icosagrid.*

Method II: Composite from E8 QCs

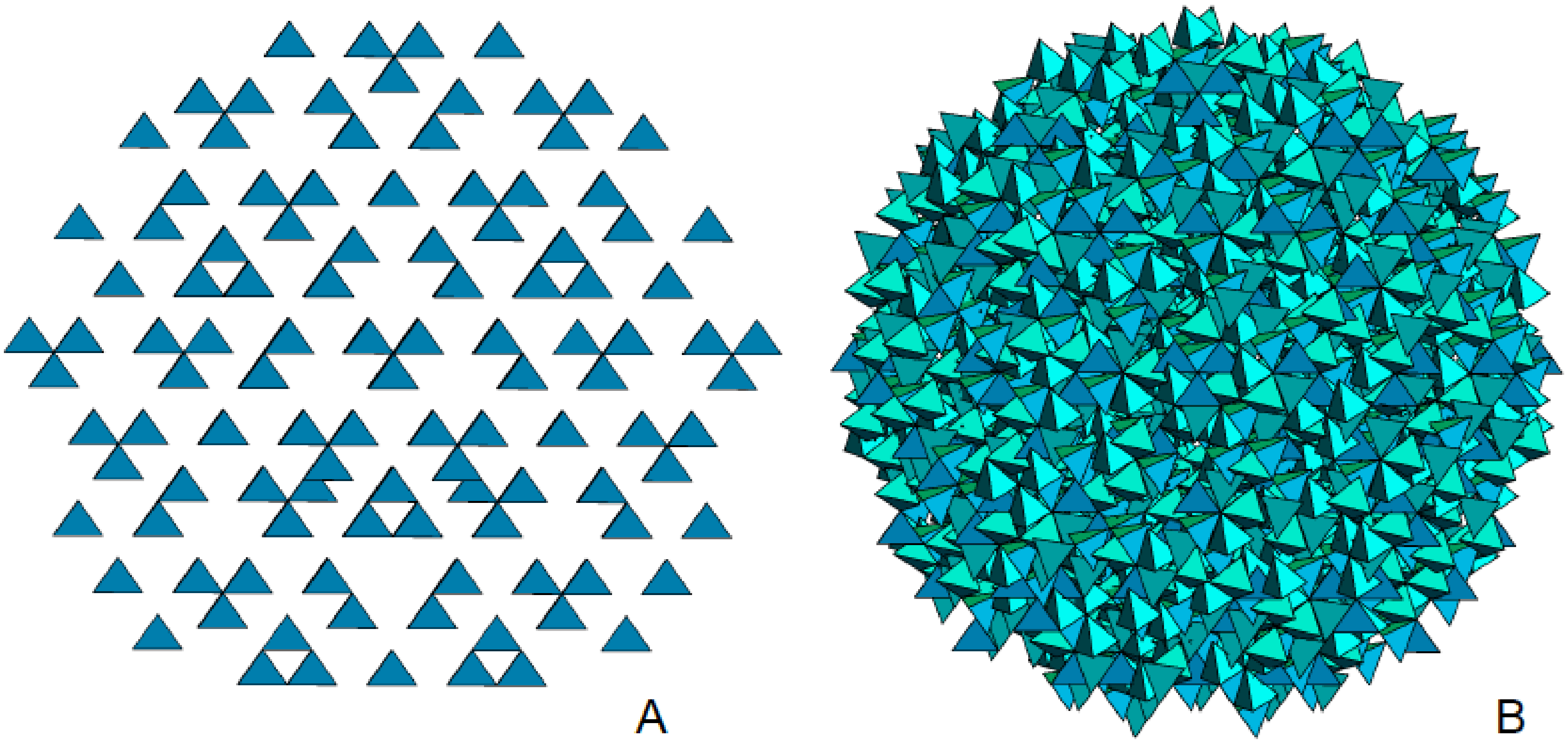


Figure 18: *(A) Type I tetrahedral cross-section of the Elser-Sloane quasicrystal, and (B) the compound quasicrystal.*

Method II: Composite from E8 QCs

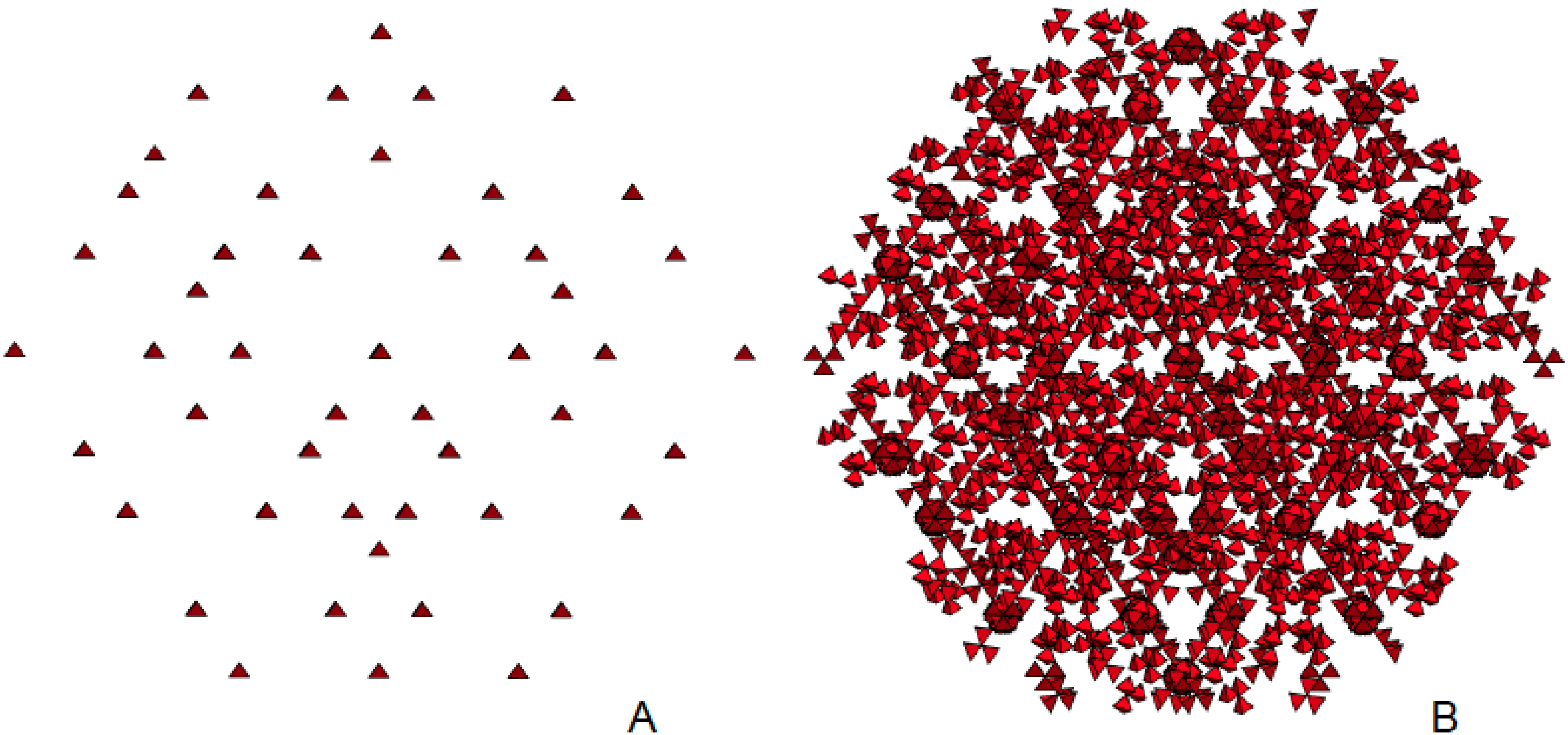


Figure 19: *(A) Type II tetrahedral cross-section with τ scaled tetrahedra, and (B) the more sparse CQC.*

Method II: Composite from E8 QCs

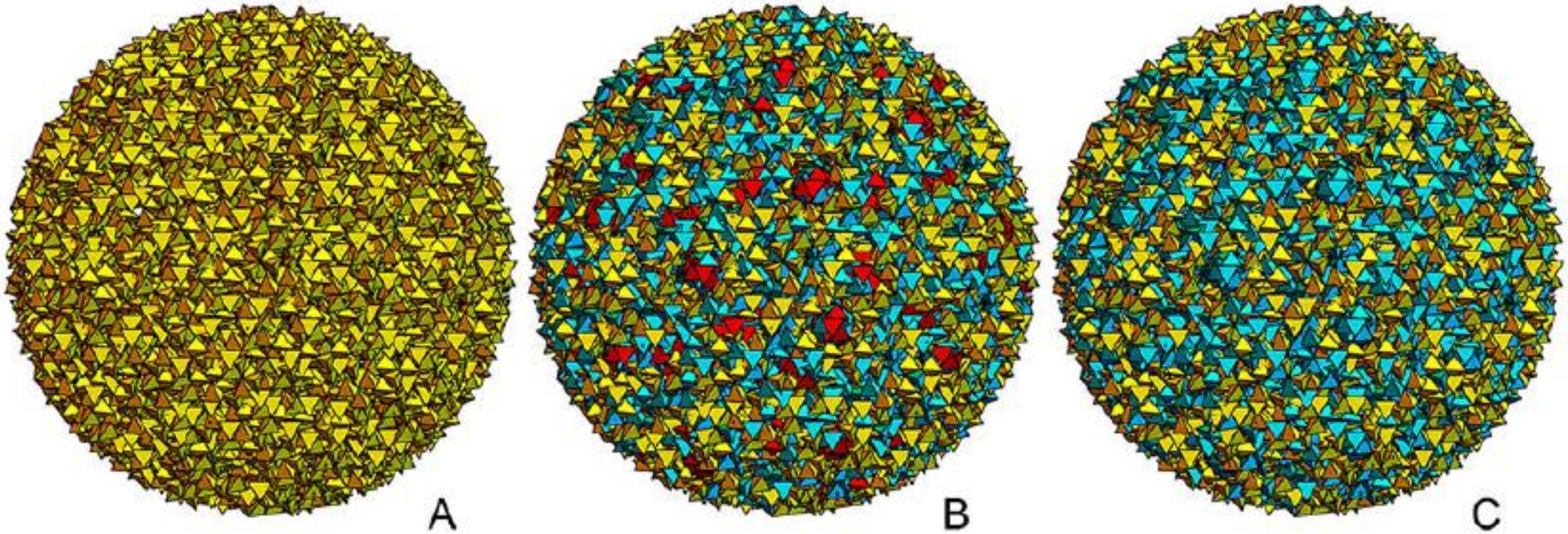
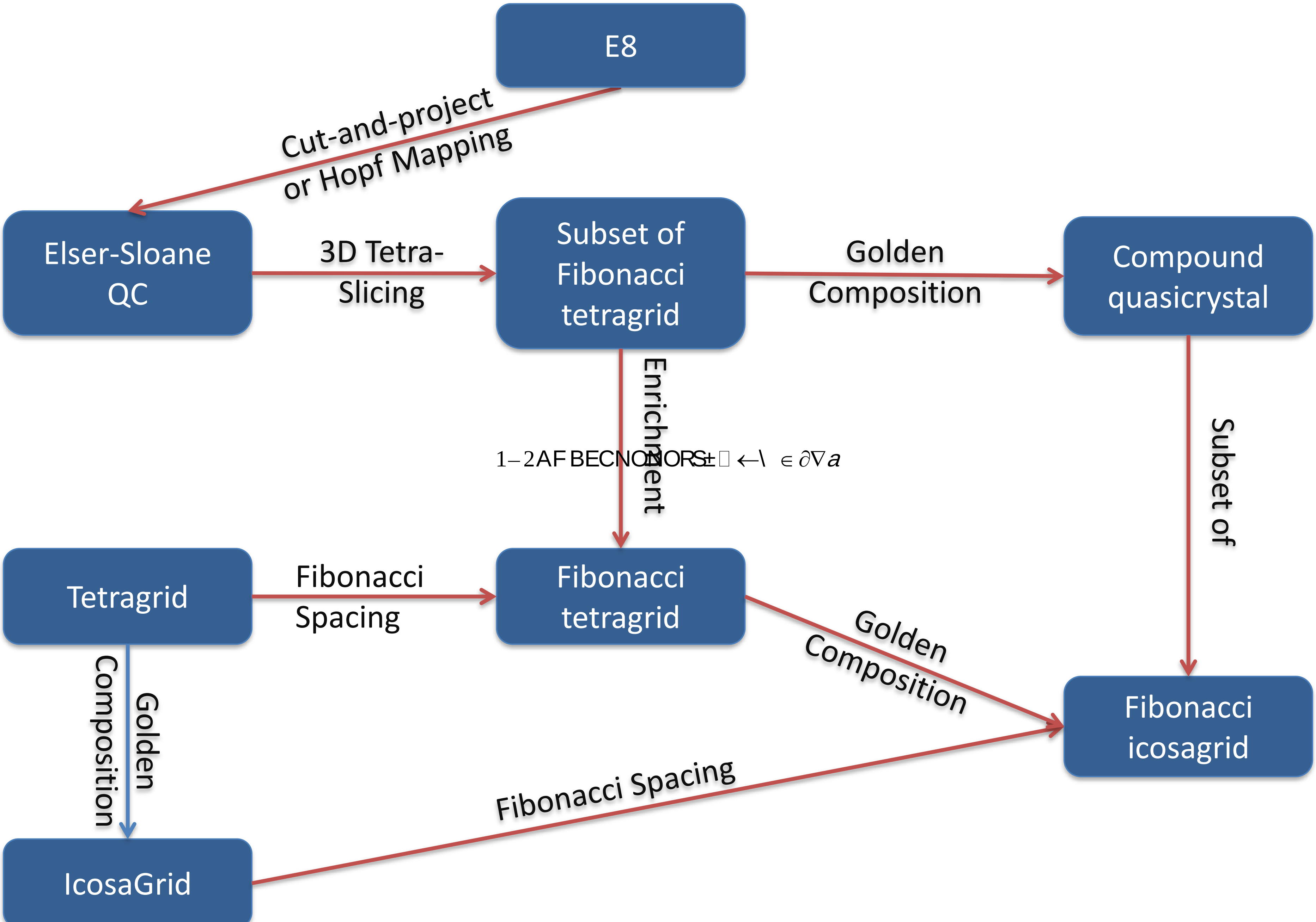


Figure 20: *(A) Tetrahedra of the Fibonacci icosagrid; (B) cyan highlighted tetrahedra belong to the Type II compound, and (C) red highlighted tetrahedra belong to the Type I compound.*



E8
Cut-and-project or Hopf Mapping
Elser-Sloane QC
3D Tetra-Slicing
Subset of Fibonacci tetragrid
Golden Composition
Compound quasicrystal
Enrichment
1−2AF BECNONORS±
Subset of
Tetragrid
Fibonacci Spacing
Fibonacci tetragrid
Golden Composition
Fibonacci icosagrid
Golden Composition
Fibonacci Spacing
IcosaGrid

Code in QSN guided from E8

- The binary on/off selection from E8 directly mapped to the on/off selection in QSN, as the possibility space.
- Ray has given other proposals on the coding in QSN

600-CELL OVERLAP TYPES