Quantum Gravity at the 5th Root of Unity

Marcelo M. Amaral

June 2018
AQG

Quantum Gravity Research
www.quantumgravityresearch.org

$SU(2)$ at 5th root of unity

- I will present here the general idea on connecting the code theoretic axiom to representation theory of Lie algebra - leading to apply Fibonacci anyons topological codes to quantum gravity and unification physics.
  (It is work in progress).

1. L. C. Biedenharn, M. A. Lohe. "Quantum group symmetry and q-tensor algebras". World Scientific (1995).

$SU(2)$ at 5th root of unity

- I will present here the general idea on connecting the code theoretic axiom to representation theory of Lie algebra - leading to apply Fibonacci anyons topological codes to quantum gravity and unification physics.
  (It is work in progress).
- First I need explain the 5th root of unity. The quantum group [1] $SU(2)$ are in fact deformations of its Lie algebra.

1. L. C. Biedenharn, M. A. Lohe. "Quantum group symmetry and q-tensor algebras". World Scientific (1995).

$SU(2)$ at 5th root of unity

- I will present here the general idea on connecting the code theoretic axiom to representation theory of Lie algebra - leading to apply Fibonacci anyons topological codes to quantum gravity and unification physics. (It is work in progress).
- First I need explain the 5th root of unity. The quantum group[1] $SU(2)$ are in fact deformations of its Lie algebra.
- These deformations depend on a deformation parameter q. When q is a real parameter, the representation theory is the same as the classical algebra. If we allow q to have arbitrary complex values, the q-deformed algebra becomes complex with non-unitary representations. However, in the special case where q is a complex root of unity there are new types of representations, which is helpful to achieve restrictions on the classical representations.

1. L. C. Biedenharn, M. A. Lohe. "Quantum group symmetry and q-tensor algebras". World Scientific (1995).

$SU(2)$ at 5th root of unity

- I will present here the general idea on connecting the code theoretic axiom to representation theory of Lie algebra - leading to apply Fibonacci anyons topological codes to quantum gravity and unification physics.
  (It is work in progress).
- First I need explain the 5th root of unity. The quantum group [1] $SU(2)$ are in fact deformations of its Lie algebra.
- These deformations depend on a deformation parameter q. When q is a real parameter, the representation theory is the same as the classical algebra. If we allow q to have arbitrary complex values, the q-deformed algebra becomes complex with non-unitary representations. However, in the special case where q is a complex root of unity there are new types of representations, which is helpful to achieve restrictions on the classical representations.
- For $SU(2)_q$, where q is a $5th$ root of unity, $q = e^{\frac{2i\pi}{5}}$, it is possible to find unitary irreducible representations which agree with the lower dimensional classical ones spin-0, spin-$\frac{1}{2}$, spin-1 and spin-$\frac{3}{2}$, being the other reducible ones.

1. L. C. Biedenharn, M. A. Lohe. "Quantum group symmetry and q-tensor algebras". World Scientific (1995).

$SU(2)$ at 5th root of unity

- The $SU(2)$ Lie algebra is given by

$$\left[J^i, J^j\right] = i\varepsilon^{ijk} J^k, \tag{1}$$

$SU(2)$ at 5th root of unity

- The $SU(2)$ Lie algebra is given by

$$\left[J^i, J^j\right] = i\varepsilon^{ijk} J^k, \tag{1}$$

- We can define raising and lowering operators

$$J_\pm = J_1 \pm iJ_2, \tag{2}$$

with

$$\begin{aligned} [J_3, J_\pm] &= \pm J_\pm \\ [J_+, J_-] &= 2J_3. \end{aligned} \tag{3}$$

(This is the usual Cartan-Weyl basis)

$SU(2)$ at 5th root of unity

- We can then introduce the deformation generator

$$\mathcal{J} = q^{J_3}. \tag{4}$$

$SU(2)_q$ algebra, which is over the complex numbers, is generated by the three operators $\mathcal{J}$, $J_\pm$ where

$$\begin{aligned} [J_+, J_-] &= 2\frac{\mathcal{J} - \mathcal{J}^{-1}}{q - q^{-1}} \\ \mathcal{J} J_\pm \mathcal{J}^{-1} &= q^{\pm 1} J_\pm, \end{aligned} \tag{5}$$

where the limit $q \to 1$ reproduces the classical algebra.

$SU(2)$ at 5th root of unity

- In contrast with classical $SU(2)$ algebra, which has representations labeled by spin quantum numbers that goes to infinity, here for the irreducible representations just the ones in a specific range of spins are admissible, namely $j = 0, \frac{1}{2}, 1, \frac{3}{2}$ and we have fusion rules :

$SU(2)$ at 5th root of unity

- In contrast with classical $SU(2)$ algebra, which has representations labeled by spin quantum numbers that goes to infinity, here for the irreducible representations just the ones in a specific range of spins are admissible, namely $j = 0, \frac{1}{2}, 1, \frac{3}{2}$ and we have fusion rules :
- $$\begin{aligned} 0 \otimes j &= j \\ \frac{3}{2} \otimes j &= \frac{3}{2} - j \\ \frac{1}{2} \otimes \frac{1}{2} &= 0 \oplus 1 \\ \frac{1}{2} \otimes 1 &= \frac{1}{2} \oplus \frac{3}{2} \\ 1 \otimes 1 &= 0 \oplus 1. \end{aligned} \tag{6}$$

$SU(2)$ at 5th root of unity

- In contrast with classical $SU(2)$ algebra, which has representations labeled by spin quantum numbers that goes to infinity, here for the irreducible representations just the ones in a specific range of spins are admissible, namely $j = 0, \frac{1}{2}, 1, \frac{3}{2}$ and we have fusion rules :
- $$\begin{aligned} 0 \otimes j &= j \\ \frac{3}{2} \otimes j &= \frac{3}{2} - j \\ \frac{1}{2} \otimes \frac{1}{2} &= 0 \oplus 1 \\ \frac{1}{2} \otimes 1 &= \frac{1}{2} \oplus \frac{3}{2} \\ 1 \otimes 1 &= 0 \oplus 1. \end{aligned} \tag{6}$$
- The quantum dimension of the representation $d_q(j)$ for $j = 0, \frac{1}{2}, 1, \frac{3}{2}$ is respectively $1, \phi, \phi, 1$, where ϕ is the golden ratio, 1.618...

$SU(2)_q$ - motivations

- Deformation is an important concept in physics : quantum mechanics is a deformation of classical mechanics, in which $q = e^{\hbar} \to 1$, that is, $\hbar \to 0$, and Einstein's relativity is a deformation of Galilean relativity, in which $q = e^{\frac{1}{c}} \to 1$, that is, $c \to \infty$. Today we are looking for a deformation of Lie groups and algebras, which allow at the same time for quantization of spacetime and unification of internal gauge symmetries.

$SU(2)_q$ - motivations

- Deformation is an important concept in physics : quantum mechanics is a deformation of classical mechanics, in which $q = e^{\hbar} \to 1$, that is, $\hbar \to 0$, and Einstein's relativity is a deformation of Galilean relativity, in which $q = e^{\frac{1}{c}} \to 1$, that is, $c \to \infty$. Today we are looking for a deformation of Lie groups and algebras, which allow at the same time for quantization of spacetime and unification of internal gauge symmetries.
- We can recover the quantum symmetry $SU(2)_q$ at the $5th$ root of unity (I will call it just $SU(2)_5$ in this presentation) in different approaches to quantum gravity and particle physics unification.

$SU(2)_q$ - motivations

- For example, in the quantization of string theory on a group manifold considering the group $SU(2)$ [2] and string-net models of gauge field emergence [3]. It appears too throughout the so-called quantum group conformal field theory duality [4] and in topological quantum field theory [5]. In quantum gravity it defines a special base for the Hilbert space of loop quantum gravity I will discuss later.

---

2. D. Gepner and E. Witten, Nucl. Phys. B **278**, 493 (1986).
3. M. A. Levin and X. G. Wen, Phys. Rev. B **71**, 045110 (2005).
4. L. Alvarez-Gaume, G. Sierra and C. Gomez, In Brink, L. (ed.) et al. : Physics and mathematics of strings 16-184 and CERN Geneva - TH. 5540 (89,rec.Mar.90) 169 p
5. E. Witten, Commun. Math. Phys. **117**, 353 (1988). E. Witten, Commun. Math. Phys. **121**, 351 (1989).

Quantum Gravity - Motivations

- Gravity and quantum, are universal phenomena. But the unified framework to understand them, the quantum gravity regime, is not completely understood yet despite the big effort of modern theoretic physics since Einstein, which point out in direction of the need for new grounds for quantum gravity.

Quantum Gravity - Motivations

- Gravity and quantum, are universal phenomena. But the unified framework to understand them, the quantum gravity regime, is not completely understood yet despite the big effort of modern theoretic physics since Einstein, which point out in direction of the need for new grounds for quantum gravity.
- The unification of forces program to understand the standard model of particle physics gauge symmetry $SU(3) \times SU(2) \times U(1)$ inside a large gauge group is not complete also.

Quantum Gravity - Motivations

- Gravity and quantum, are universal phenomena. But the unified framework to understand them, the quantum gravity regime, is not completely understood yet despite the big effort of modern theoretic physics since Einstein, which point out in direction of the need for new grounds for quantum gravity.
- The unification of forces program to understand the standard model of particle physics gauge symmetry $SU(3) \times SU(2) \times U(1)$ inside a large gauge group is not complete also.
- There are different proposals with large Lie groups, $SU(5)$, $SO(10)$, E_6, E_8 for example, which allows for classical formulations, even providing Lagrangians. But yet there are no consensus and the quantization seems very hard. The simplest one, $SU(5)$, where it is possible to do the path integral quantization in more details, has predictions not confirmed and some fundamental issues for example (proton decay, baryon and lepton-number violating processes, Higgs triplet hierarchy problem, incorrect prediction of Weinberg mixing angle)

Quantum Gravity - Motivations

- The approaches to the two problems are convergent. If we unify particle/field forces at unification scale we can ask about including gravity, going in direction to the Planck scale. Quantum gravity approaches like string theory and loop quantum gravity (LQG) propose to start direct from quantum gravity regime with new principles and assumptions and to recover unification physics at low energy, which create a gap between theory and experiment but give us new insights.

Quantum Gravity - Motivations

- The approaches to the two problems are convergent. If we unify particle/field forces at unification scale we can ask about including gravity, going in direction to the Planck scale. Quantum gravity approaches like string theory and loop quantum gravity (LQG) propose to start direct from quantum gravity regime with new principles and assumptions and to recover unification physics at low energy, which create a gap between theory and experiment but give us new insights.
- For example we have deep ideas not supported by experiments : a symmetry to unify fermions and bosons, higher spacetime dimensions, large dimensional particles and discretization of spacetime (quantization of geometry).

Quantum Gravity - Motivations

- The approaches to the two problems are convergent. If we unify particle/field forces at unification scale we can ask about including gravity, going in direction to the Planck scale. Quantum gravity approaches like string theory and loop quantum gravity (LQG) propose to start direct from quantum gravity regime with new principles and assumptions and to recover unification physics at low energy, which create a gap between theory and experiment but give us new insights.
- For example we have deep ideas not supported by experiments : a symmetry to unify fermions and bosons, higher spacetime dimensions, large dimensional particles and discretization of spacetime (quantization of geometry).
- From strings and LQG, in language of quantum mechanics, it is conjectured that quantum gravity regime involves quantum amplitudes for geometries and particle/field in the same object – transition amplitudes.

Quantum Gravity - A new principle

- Physical principles, inferred from observed phenomena, serve as a roadmap in constructing fundamental physical theories. For example, the equivalence principle was an important tool in the development of general relativity. The uncertainty principle discovered by Heisenberg helped in the development of quantum mechanics.

Quantum Gravity - A new principle

- Physical principles, inferred from observed phenomena, serve as a roadmap in constructing fundamental physical theories. For example, the equivalence principle was an important tool in the development of general relativity. The uncertainty principle discovered by Heisenberg helped in the development of quantum mechanics.
- The holographic principle is meaningful because it arises in this context of this problem of quantizing spacetime and gravity along with the quest for unification of fundamental quantum fields. The holographic principle was proposed from logical considerations of physical phenomena associated with gravitational collapse and lead to the idea that physics at the Planck scale is constrained to be lower dimensional and possess finite degrees of freedom. At the same time, it is fundamentally a different kind of principle because it is not directly supported by experiments.

Code theoretic principle

- We discuss a principle for quantum gravity that is deeply correlated to the Lie algebraic basis of particle physics derived from particle accelerator experiments in order to non-arbitrarily derive our Planck scale restrictions and degrees of freedom.

6. K. Irwin, "The Code-Theoretic Axiom". FQXI Essay Contest (2017), http ://fqxi.org/community/forum/topic/2901.

Code theoretic principle

- We discuss a principle for quantum gravity that is deeply correlated to the Lie algebraic basis of particle physics derived from particle accelerator experiments in order to non-arbitrarily derive our Planck scale restrictions and degrees of freedom.
- The new ontological start point is that at fundamental level nature is neither random nor deterministic, but code theoretic [6].

6. K. Irwin, "The Code-Theoretic Axiom". FQXI Essay Contest (2017), http ://fqxi.org/community/forum/topic/2901.

Code theoretic principle

- We discuss a principle for quantum gravity that is deeply correlated to the Lie algebraic basis of particle physics derived from particle accelerator experiments in order to non-arbitrarily derive our Planck scale restrictions and degrees of freedom.
- The new ontological start point is that at fundamental level nature is neither random nor deterministic, but code theoretic [6].
- The precise meaning of a physical spatiotemporal code is : *(1) A finite set of symbolic object, (2) ordering rules and (3) syntactical freedom, (4) for the purpose of expressing meaning, i.e., self-referential physical meaning*.

6. K. Irwin, "The Code-Theoretic Axiom". FQXI Essay Contest (2017), http ://fqxi.org/community/forum/topic/2901.

Quantum Gravity - Code theoretic

- Let us clarify the code theoretic concept considering anyonic topological codes :

7. Zhenghan Wang. *Topological Quantum Computation*; Number 112. American Mathematical Soc., 2010. Jiannis K. Pachos. *Introduction to Topological Quantum Computation*; Cambridge University Press, 2012

Quantum Gravity - Code theoretic

- Let us clarify the code theoretic concept considering anyonic topological codes :
- To describe a system of anyons, the usual approach is to list the species of anyons in the system, also called the particle types, topological charges or simply labels. These are the “letters” or the finite set of symbolic object types of this topological code.

7. Zhenghan Wang. *Topological Quantum Computation*; Number 112. American Mathematical Soc., 2010. Jiannis K. Pachos. *Introduction to Topological Quantum Computation*; Cambridge University Press, 2012

Quantum Gravity - Code theoretic

- Let us clarify the code theoretic concept considering anyonic topological codes :
- To describe a system of anyons, the usual approach is to list the species of anyons in the system, also called the particle types, topological charges or simply labels. These are the “letters” or the finite set of symbolic object types of this topological code.
- Then there are the so called fusion rules, which specify how these fundamental labels can be coupled. These rules are not deterministic and depending on the class of anyon we are dealing with.

7. Zhenghan Wang. *Topological Quantum Computation* ; Number 112. American Mathematical Soc., 2010. Jiannis K. Pachos. *Introduction to Topological Quantum Computation* ; Cambridge University Press, 2012

Quantum Gravity - Code theoretic

- Let us clarify the code theoretic concept considering anyonic topological codes :
- To describe a system of anyons, the usual approach is to list the species of anyons in the system, also called the particle types, topological charges or simply labels. These are the "letters" or the finite set of symbolic object types of this topological code.
- Then there are the so called fusion rules, which specify how these fundamental labels can be coupled. These rules are not deterministic and depending on the class of anyon we are dealing with.
- The last component of a code is its function to express meaning like for example, to express a specific quantum computation. It is well known that some anyonic systems can perform universal quantum computation, such as the simple non abelian anyon – the Fibonacci anyon [7].

7. Zhenghan Wang. *Topological Quantum Computation* ; Number 112. American Mathematical Soc., 2010. Jiannis K. Pachos. *Introduction to Topological Quantum Computation* ; Cambridge University Press, 2012

Fibonacci anyon

- Fibonacci anyon fusion rules are irreducibly simple and can be understood in terms of the representation theory of quantum groups, especially the quantum $SU(2)_5$ discussed in the beginning.
  The fusion rules for Fibonacci anyons are in this case :

$$\begin{aligned} 1 \otimes 1 &= 0 \oplus 1 \\ 0 \otimes 1 &= 1 \\ 1 \otimes 0 &= 1 \end{aligned} \tag{7}$$

  where we make use of the label spin to express the representations.

Fibonacci anyon and efficient codes

- The idea of a code brings the requirement of efficiency. The same $SU(2)_r$ above is used to describe different topological phases of matter (with different anyons type). Each root r describe one topological phase. If we rank the topological phases associated with non-abelian anyons described by $SU(2)_r$ in terms of topological order defined as the inverse of topological entanglement entropy [8], γ, the Fibonacci anyons represent a limit in terms of higher topological order. Another limit is the minimum 2 letters and simple fusion rules in the family on non abelian anyons.

$$\gamma = log\mathcal{D}$$

where where $\mathcal{D} \geq 1$ is the total quantum dimension. In $SU(2)_r$ theory :

$$\mathcal{D} = \frac{\sqrt{r/2}}{sin(\pi/r)}$$

8. A. Kitaev and J. Preskill, Phys. Rev. Lett. **96**, 110404 (2006), [hep-th/0510092].

Covariant Loop Quantum Gravity

- One starts from a gravity action in terms of triads/tetrads, do the discretization in a two-complex, the dual of a triangulation : general relativity can be truncated on a two-complex by associating an $SU(2)$ group element U_e to each edge e and an $su(2)$ algebra element L_f to each face f of the dual triangulation.

9. C. Rovelli ; F. Vidotto, "Covariant Loop Quantum Gravity", Cambridge University Press, 2014.

Covariant Loop Quantum Gravity

- One starts from a gravity action in terms of triads/tetrads, do the discretization in a two-complex, the dual of a triangulation : general relativity can be truncated on a two-complex by associating an $SU(2)$ group element U_e to each edge e and an $su(2)$ algebra element L_f to each face f of the dual triangulation.

- The quantization of a phase space which is the cotangent space to a group is standard [9]. We are seeking operators U_l and L_l realizing a commutation relation on a Hilbert space.

$$\left[U_l, L^i_{l'}\right] = i\beta\delta_{ll'} U_l J^i$$

$$\left[L^i_l, L^j_{l'}\right] = i\beta\delta_{ll'}\varepsilon^{ij}_k L^k_l$$

with

$$\mathcal{H}_\Gamma = L_2\left[SU(2)^L\right]$$

9. C. Rovelli ; F. Vidotto, "Covariant Loop Quantum Gravity", Cambridge University Press, 2014.

Covariant Loop Quantum Gravity

- The results can be summarized as : A basis for the Hilbert space is given by $SU(2)$ spin network states, graphs Γ colored by spins j_l on every link l, and a set of intertwiner labels i_l on every node, which implement the fusion rules of $SU(2)$.

Covariant Loop Quantum Gravity

- The results can be summarized as : A basis for the Hilbert space is given by $SU(2)$ spin network states, graphs Γ colored by spins j_l on every link l, and a set of intertwiner labels i_l on every node, which implement the fusion rules of $SU(2)$.
- For the evolution of Γ we consider an open 2-complex C whose boundary is Γ. It consists of vertices, edges and faces, and to each node and link of Γ there correspond a unique edge and face of the boundary of C. We color it with spins representations j_f the faces and intertwiners i_e the edges.

Covariant Loop Quantum Gravity

- The evolution of this spin network states is given by transition amplitudes

$$W(j_l) = w \sum_{j_f} \prod_{f,e} A_{j_f e} \prod_{v} A_v(j_f) \tag{8}$$

Covariant Loop Quantum Gravity

- The evolution of this spin network states is given by transition amplitudes

$$W(j_l) = w \sum_{j_f} \prod_{f,e} A_{j_f e} \prod_{v} A_v(j_f) \tag{8}$$

- The vertex amplitudes $A_v(j_f)$ are model dependent. For example, if it is 3D or 4D, Euclidean or Lorentzian, classical or quantum symmetry.

Covariant Loop Quantum Gravity

- The evolution of this spin network states is given by transition amplitudes

$$W(j_l) = w \sum_{j_f} \prod_{f,e} A_{j_f e} \prod_{v} A_v(j_f) \tag{8}$$

- The vertex amplitudes $A_v(j_f)$ are model dependent. For example, if it is 3D or 4D, Euclidean or Lorentzian, classical or quantum symmetry.

- But, interestingly, it doesn't matter if it is 3D or 4D, Euclidean or Lorentzian, the kinematics, doesn't change, the Hilbert space can be described by the same $SU(2)$ spin network. What change is the dynamics, the vertex amplitudes. This allows to implement the code theoretic principle discussed before.

LQG at 5th root of unity

- We have a proposal for 3D considering evolution of spin networks restricted to representations of $SU(2)_5$:

LQG at 5th root of unity

- We have a proposal for 3D considering evolution of spin networks restricted to representations of $SU(2)_5$:
-

$$W_q(j_l) = w_q \sum_{j_f} \prod_f d_q(j_f) \prod_v \{6j\}_q \tag{9}$$

where for $r = 5$ and $q = e^{\frac{2i\pi}{r}}$, $d_q(j)$ for $j = 0, \frac{1}{2}, 1, \frac{3}{2}$ is respectively $1, \phi, \phi, 1$

$$\{6j\}_q = \left\{ \begin{array}{ccc} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{array} \right\}_q \tag{10}$$

which is the Wigner 6j-symbol.
The q-deformed 6_j symbols for every combination of js in $\{0,1/2,1,3/2\}$ has most of the possibilities null, except a few, which take only seven values : $\{0, \pm 1, \pm\phi, \pm i\sqrt{\phi}\}$.

Semi-classical limit

- The vertex amplitude given by quantum 6j-symbol approaches a path integral for general relativity with cosmological constant [10] in the limit of large r and large quantum numbers j, which indicate it is in the write direction.

$$\begin{Bmatrix} rj_1 & rj_2 & rj_3 \\ rj_4 & rj_5 & rj_6 \end{Bmatrix}_q \underset{r\to\infty}{\sim} \sqrt{\frac{1}{r^3}} cos\left(S + \frac{\pi}{4}\right)$$

where S is the Einstein–Hilbert action with cosmological constant.

10. C. Rovelli ; F. Vidotto, "Covariant Loop Quantum Gravity," Cambridge University Press, 2014.

Semi-classical limit

- The vertex amplitude given by quantum 6j-symbol approaches a path integral for general relativity with cosmological constant [10] in the limit of large r and large quantum numbers j, which indicate it is in the write direction.

$$\left\{ \begin{array}{ccc} rj_1 & rj_2 & rj_3 \\ rj_4 & rj_5 & rj_6 \end{array} \right\}_q \underset{r\to\infty}{\sim} \sqrt{\frac{1}{r^3}} cos\left(S + \frac{\pi}{4}\right)$$

where S is the Einstein–Hilbert action with cosmological constant.

- But this is not very helpful because a formal path integral for GR would be very hard to compute and we want to understand how nature compute with its underlying Planck scale code. Like I talked before, it is conjectured that quantum gravity regime involves quantum amplitudes for geometries and particle/field in the same object. So now we explore the particle/field side of the amplitude. And start making conjectures on how to fill the gap between quantum gravity regime and unification of quantum fields.

10. C. Rovelli ; F. Vidotto, "Covariant Loop Quantum Gravity," Cambridge University Press, 2014.

Representations of Lie groups and quantum fields

- The quantum fields of the standard model of particle physics observed in particles accelerators are defined in flat Minkowski space-time whose group of symmetries is the Poincaré group.

Representations of Lie groups and quantum fields

- The quantum fields of the standard model of particle physics observed in particles accelerators are defined in flat Minkowski space-time whose group of symmetries is the Poincaré group.
- It is associated spin quantum number with the Lorentz subgroup of rotations $SO(1,3)$ and mass quantum numbers with the subgroup of translations of the Poincaré group. Charge (electric, color...) is associated with internal (gauge) groups of symmetry $SU(3) \times SU(2) \times U(1)$.

Quantum Poincaré group at the 5*th* root of unity

- So, what are the symmetries of flat quantum space-time ?
  With focus on spin, the concept of relativistic fields is that they are finite representations of the Lorentz group. Allowing for a complexification of this group we can use the quantum $SU(2)_5$ to address the quantum $SO(1,3)$.

Quantum Poincaré group at the 5*th* root of unity

- So, what are the symmetries of flat quantum space-time ?
  With focus on spin, the concept of relativistic fields is that they are finite representations of the Lorentz group. Allowing for a complexification of this group we can use the quantum $SU(2)_5$ to address the quantum $SO(1,3)$.

- A general matrix Λ^{μ}_{ν} of $SO(1,3)$ can be written as

$$\Lambda(w) = e^{\frac{1}{2} w^{\alpha\beta} \Sigma_{\alpha\beta}} \tag{11}$$

where $w^{\alpha\beta}$ are infinitesimal parameters and $\Sigma_{\alpha\beta}$ the generators.

Quantum Poincaré group at the 5th root of unity

- We can rewrite the generators $\Sigma_{\alpha\beta}$ in terms of generators of two independent $SU(2)$ subalgebras in the standard way (with $M^i = \frac{1}{2}\varepsilon^{ijk}\Sigma_{jk}$, $N^i = \Sigma^{0i}$ and $J^i = \frac{i}{2}\left(M^i + iN^i\right)$, $G^i = \frac{i}{2}\left(M^i - iN^i\right)$) :

$$\left[J^i, J^j\right] = i\varepsilon^{ijk}J^k,$$
$$\left[G^i, G^j\right] = i\varepsilon^{ijk}G^k,$$
$$\left[J^i, G^i\right] = 0, \qquad (12)$$

J^i and G^i obey $SU(2)$ Lie algebra commutation relations and the Lorentz group representation can be written from these two complex $SU(2)$ representations with independent generators J^i, G^i. $SO(1,3)$ decomposes, as a direct sum, to

$$SO(1,3) = SU(2)_J \oplus SU(2)_G. \qquad (13)$$

Quantum Poincaré group at the 5th root of unity

- In particular, for each subalgebra $SU(2)$ there is a Casimir operator, $J^i J^i$, $G^i G^i$, commuting with each element of the algebra and with eigenvalues $j(j+1)$, $g(g+1)$ ($j = g = 0, \frac{1}{2}, 1, \frac{3}{2}, 2...$). Being invariants, its eigenvalues are conserved, so they provide good quantum numbers to index the representations of $SO(1,3)$ by pairs (j,g) with eigenvalues $j(j+1)$ and $g(g+1)$. The total spin of the representation (j,g) is given by $s = j + g$ and its dimension by $dim(j,g) = (2j+1)(2g+1)$. The low dimensional irreducible representations of the Lorentz group are used to model quantum fields.

Quantum Poincaré group at the 5*th* root of unity

- Now we can consider the quantum version of each $SU(2)$ from the beginning of the presentation to get code theoretic restrictions on $SO(1,3)$

Quantum Poincaré group at the 5*th* root of unity

- Now we can consider the quantum version of each $SU(2)$ from the beginning of the presentation to get code theoretic restrictions on $SO(1,3)$
-

$$
\begin{aligned}
0 \otimes j &= j \\
\frac{3}{2} \otimes j &= \frac{3}{2} - j \\
\frac{1}{2} \otimes \frac{1}{2} &= 0 \oplus 1 \\
\frac{1}{2} \otimes 1 &= \frac{1}{2} \oplus \frac{3}{2} \\
1 \otimes 1 &= 0 \oplus 1.
\end{aligned} \tag{14}
$$

Quantum Poincaré group at the 5th root of unity

- Now we can consider the quantum version of each $SU(2)$ from the beginning of the presentation to get code theoretic restrictions on $SO(1,3)$
-

$$
\begin{aligned}
0 \otimes j &= j \\
\frac{3}{2} \otimes j &= \frac{3}{2} - j \\
\frac{1}{2} \otimes \frac{1}{2} &= 0 \oplus 1 \\
\frac{1}{2} \otimes 1 &= \frac{1}{2} \oplus \frac{3}{2} \\
1 \otimes 1 &= 0 \oplus 1.
\end{aligned} \tag{14}
$$

- So the quantum spin network that is a base for the Hilbert space of quantum gravity can be understood as interaction network of low dimensional quantum Lorentzian representations.

Observables

- So we can use transition amplitudes for quantum geometry

$$W_q(j_l) = w_q \sum_{j_f} \prod_f d_q(j_f) \prod_v \{6j\}_q \tag{15}$$

to compute not only geometric observables but matter observable also.

Observables

- So we can use transition amplitudes for quantum geometry

$$W_q(j_l) = w_q \sum_{j_f} \prod_f d_q(j_f) \prod_v \{6j\}_q \tag{15}$$

to compute not only geometric observables but matter observable also.

- The transition between two spin network states can be interpreted as an expected value of observable

$$W_{\mathcal{O}}(\Gamma_{j_f}) = \sum_{j_i} W_q(j_i, j_f)\Gamma_{j_i}$$

Observables

- So we can use transition amplitudes for quantum geometry

$$W_q(j_l) = w_q \sum_{j_f} \prod_f d_q(j_f) \prod_v \{6j\}_q \quad (15)$$

to compute not only geometric observables but matter observable also.

- The transition between two spin network states can be interpreted as an expected value of observable

$$W_{\mathcal{O}}(\Gamma_{j_f}) = \sum_{j_i} W_q(j_i, j_f)\Gamma_{j_i}$$

- Or we can consider an observable to be a graph with edges labeled j_e.

$$W_{\mathcal{O}}(j_e) = \sum_{j_i | j_e} W_q(j_i)$$

Full quantum Poincaré group at the 5*th* root of unity

- For the Poincaré group as a whole we need to include the generators of spacetime translations P^{μ} together with the Lorentz generators $\Sigma_{\mu\nu}$ and the algebra is

$$
\begin{aligned}
[P_{\mu}, P_{\nu}] &= 0, \\
[\Sigma_{\alpha\beta}, P_{\mu}] &= -i\left(g_{\alpha\mu}P_{\beta} - g_{\beta\mu}P_{\alpha}\right), \\
[\Sigma_{\alpha\beta}, \Sigma_{\mu\nu}] &= \left(g_{\alpha\nu}\Sigma_{\beta\mu} + g_{\beta\mu}\Sigma_{\alpha\nu} - g_{\alpha\mu}\Sigma_{\beta\nu} - g_{\beta\nu}\Sigma_{\alpha\mu}\right).
\end{aligned} \tag{16}
$$

11. J. Lukierski, "Kappa-deformations : Historical Developments and Recent Results", arXiv : 1611.10213.

Full quantum Poincaré group at the 5*th* root of unity

- For the Poincaré group as a whole we need to include the generators of spacetime translations P^μ together with the Lorentz generators $\Sigma_{\mu\nu}$ and the algebra is

$$\begin{aligned} [P_\mu, P_\nu] &= 0, \\ [\Sigma_{\alpha\beta}, P_\mu] &= -i\left(g_{\alpha\mu}P_\beta - g_{\beta\mu}P_\alpha\right), \\ [\Sigma_{\alpha\beta}, \Sigma_{\mu\nu}] &= \left(g_{\alpha\nu}\Sigma_{\beta\mu} + g_{\beta\mu}\Sigma_{\alpha\nu} - g_{\alpha\mu}\Sigma_{\beta\nu} - g_{\beta\nu}\Sigma_{\alpha\mu}\right). \end{aligned} \tag{16}$$

- The q-deformation scheme presented for $SU(2)_5$ works for complex Lie algebras. For the real Poincaré algebras the main approach is to start with a deformation of the conformal algebra and group $SO(4,2)$ and after this, to perform a contraction to Poincaré, the so called kappa-Poincaré algebra [11]. We are starting study the 5*th* root of unity case with this deformation, which should provide the mass quantum number for relativistic particles.

11. J. Lukierski, "Kappa-deformations : Historical Developments and Recent Results", arXiv : 1611.10213.

Charge and E8 Lie group at the 5*th* root of unity

- The procedure to diagonalise one generator of $SU(2)$, usually J_3 and then to re-express the other operators in terms of eigenoperators of the chosen diagonal operator,

$$J_\pm = J_1 \pm iJ_2, \tag{17}$$

with

$$\begin{aligned} [J_3, J_\pm] &= \pm J_\pm \\ [J_+, J_-] &= 2J_3. \end{aligned} \tag{18}$$

can be generalized to large dimensional Lie algebras. The Cartan-Weyl basis.

Charge and E8 Lie group at the 5*th* root of unity

- The Cartan-Weyl basis may be written as

$$[H_i, H_j] = 0$$
$$[H_i, E_\alpha] = \alpha_i E_\alpha$$
$$[E_\alpha, E_\beta] = N_{\alpha\beta} E_{\alpha+\beta}$$

where H_i are the generators of the Cartan subalgebra and E_α the root vectors.

Charge and E8 Lie group at the 5*th* root of unity

- The Cartan-Weyl basis may be written as

$$
\begin{aligned}
[H_i, H_j] &= 0 \\
[H_i, E_\alpha] &= \alpha_i E_\alpha \\
[E_\alpha, E_\beta] &= N_{\alpha\beta} E_{\alpha+\beta}
\end{aligned}
$$

  where H_i are the generators of the Cartan subalgebra and E_α the root vectors.

- So the representation theory can be understood in terms of interactions of $SU(2)$ representations, and $SU(2)_5$ quantization can help understand the main role of low dimensional (fundamental and adjoint) representations to characterize charge quantum numbers.

Quasicrystalline Spin Network

- We are investigating the quasicrystalline representation of unification groups and its correlation with the quantization here described. The root lattice of the unification groups $SU(5)$, $SO(12)$ and $E8$, which encode information of representations of this Lie groups, can be projected to the quasicrystals associated with the non crystallographic Coxeter groups $H2$, $H3$ and $H4$ respectively. The cut-and-project method used to do the projection can be used to project only the lower dimensional representations of this groups in a code theoretic way.

Quasicrystalline Spin Network

- We are investigating the quasicrystalline representation of unification groups and its correlation with the quantization here described. The root lattice of the unification groups $SU(5)$, $SO(12)$ and $E8$, which encode information of representations of this Lie groups, can be projected to the quasicrystals associated with the non crystallographic Coxeter groups $H2$, $H3$ and $H4$ respectively. The cut-and-project method used to do the projection can be used to project only the lower dimensional representations of this groups in a code theoretic way.

- Another research is about quantum amplitudes and transition amplitudes, which is an interaction network of quantum dimensions that can be generated by the cut and project method. A clue for this is that the R matrix of $SU(2)_5$, its centralizer, can be used to generate non crystallographic braid groups. So there are a quasicrystalline centralizer code within $SU(2)_5$. The evolution of this qusicrystalline spin network is suitable to simulate with quantum walk, cellular automata, game of life type approaches. Classical $SU(2)$ is addressed in lattice gauge simulations with Monte Carlos techniques for example.

Quantum Gravity at 5th Root of Unity

Thank you.