Quantum walk on spin network

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Background/Overview

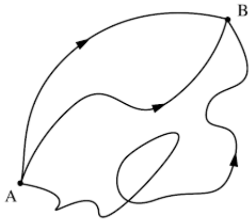
$\rightarrow$ Quantum gravity by usual Feynman Path Integral
$\rightarrow$ Loop quantum gravity(LQG): Spin Foam models
$\rightarrow$ In LQG, spin networks define quantum states of the gravitational field
$\rightarrow$ Consider a quantum particle on this gravitational field
$\rightarrow$ Hamiltonian
$\rightarrow$ Transition probabilities
$\rightarrow$ Randon walk
$\rightarrow$ Quantum walk

Motivations

→ Feynman path integral quantization
Transition amplitudes

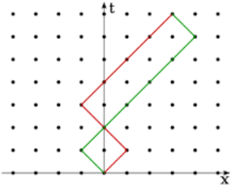
$$W = \int D[]e^{\frac{i}{h}S} \tag{1}$$

→ Sum over histories



Motivations

→ Feynman checkerboard → quantum random walk



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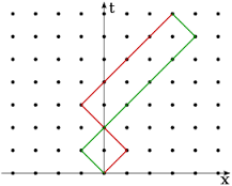

→ It from bit
→ From an ontological viewpoint we will see that dynamics and mass emerge from the spin network topology, and that quantum walk implements it.

Spin network

$\rightarrow$ Spin network graph $\Gamma = (V(\Gamma), L(\Gamma))$,
with $V(\Gamma) \rightarrow v_1, v_2...$, vertices
and $L(\Gamma) \rightarrow l_1, l_2...$, $(\{\frac{1}{2}, 1, \frac{3}{2}...\})$ links

$\rightarrow$ For the particle state space the relevant contribution comes from the position on the vertices of graph Γ.

$\rightarrow$ Hamiltonian operator on a fixed spin network [1]

$$\langle\psi|H|\psi\rangle = \kappa \sum_l j_l(j_l+1)\left(\psi(l_f) - \psi(l_i)\right)^2, \qquad (2)$$

l_f are the final points of the link l and l_i the initial points

[1] C. Rovelli; F. Vidotto, Phys. Rev. D 2010, 81, arXiv:0905.2983v2.

Transition probabilities

→ If the link l starts at vertex m and ends at vertex n we can change the notation, relabelling the color of this link l between m and n as j_{mn} and the wave function on the end points as $\psi(v_m)$, $\psi(v_n)$. Now, for H, we have

$$\langle\psi|H|\psi\rangle = \kappa \sum_l j_{mn}(j_{mn}+1)\left(\psi(v_n)-\psi(v_m)\right)^2 , \qquad (3)$$

→ Random walk associated with (3) have transition probabilities [2]

$$P_{mn} = \frac{j_{mn}(j_{mn}+1)}{\sum_k j_{mk}(j_{mk}+1)} . \qquad (4)$$

[2] J. M. Garcia-Islas, ARXIV gr-qc/1411.4383.

Quantum walk

$\rightarrow$ Consider the N_v-dimensional Hilbert space
$\mathcal{H}_n$, $\{|n\rangle, n = 1, 2, ..., N_v\}$ and
$\mathcal{H}_m$, $\{|m\rangle, m = 1, 2, ..., N_v\}$,
where N_v is the number of the vertex $V(\Gamma)$.

$\rightarrow$ The state of the walk is given in the product $\mathcal{H}_n^{N_v} \otimes \mathcal{H}_m^{N_v}$ spanned by these bases, by states at the previous $|m\rangle$ and current $|n\rangle$ steps, defined by [3]

$$|\psi_n(t)\rangle = \sum_m^{N_v} \sqrt{P_{mn}}\, |n\rangle \otimes |m\rangle \,, \tag{5}$$

[3] M. Szegedy, arXiv:quant-ph/0401053, arXiv:quant-ph/0401053v11.

Quantum walk

$\rightarrow$ For the evolution define a reflection, which we can interpret with the “coin” operator

$$C = 2\sum_{n} |\psi_n\rangle \langle\psi_n| - I, \tag{6}$$

and a swap operation

$$S = \sum_{n,m} |m, n\rangle \langle n, m| \,, \tag{7}$$

and we have the unitary evolution

$$U = CS, \tag{8}$$

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→ It is given by equations (5 - 8) with P given by (4). Namely, for equation (5)

$$|\psi_n(t)\rangle = \sum_{m}^{N_v} \sqrt{\frac{j_{mn}(j_{mn}+1)}{\sum_k j_{mk}(j_{mk}+1)}}\, |n\rangle \otimes |m\rangle \, . \quad (9)$$

Quantum walk

→ To do:

- Continuum limit;

- Two particle QW and entanglement;

- Others Laplacians [4].

[4] Johannes Thurigen tesis, arXiv:1510.08706v1

Entanglement Entropy

$\rightarrow$ Consider the Schmidt decomposition. Take a Hilbert space $\mathcal{H}$ and decompose it into two subspaces $\mathcal{H}_1$ of dimension N_1 and $\mathcal{H}_2$ of dimension $N_2 \geq N_1$, so

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2. \tag{10}$$

Let $|\psi\rangle \in \mathcal{H}_1 \otimes \mathcal{H}_2$, and $\{|\psi_i^1\rangle\} \subset \mathcal{H}_1$, $\{|\psi_i^2\rangle\} \subset \mathcal{H}_2$, and positive real numbers $\{\lambda_i\}$, then the Schmidt decomposition read

$$|\psi\rangle = \sum_i^{N_1} \sqrt{\lambda_i} \left|\psi_i^1\right\rangle \otimes \left|\psi_i^2\right\rangle, \tag{11}$$

where λ_i are the Schmidt coefficients and the number of the terms in the sum is the Schmidt rank, which we label N.
$\rightarrow$ With this we can calculate the Entanglement Entropy between the two subspaces

$$S_E = -\sum_{i \in N} \lambda_i log \lambda_i. \tag{12}$$

Entanglement Entropy

→ We can now calculate the local Entanglement Entropy between the previous step and the current n step (similar for current and next). Identify the Schmidt coefficients λ_i with our P_{mn}. Note that the Schmidt rank N is the valence of the vertex. Then the local Entanglement Entropy on current step is

$$S_{E_n} = -\sum_{m}^{N} P_{mn} log P_{mn}. \tag{13}$$

By maximizing Entanglement Entropy

$$S_{E_n} = log N_{max}, \tag{14}$$

where N_{max} is the largest valence.

Entropic motion

→ For particular cases the particle will move to a place where the entropy is the largest. This can be though of as an entropic motion.

→ And we can compute the change of entropy with respect to position for this motion. In our case as we have a discrete system we have that the variation of entropy with respect to position is just a difference of the local entropies at neighbour vertices

$\frac{dS}{dx} = |S_{E_n} - S_{E_m}|$

proportional to a small number identified with the particle mass M [5]

$$\frac{dS}{dx} = |S_{E_n} - S_{E_m}| = \alpha M, \tag{15}$$

where α is a constant of dimension $[bit/mass]$.

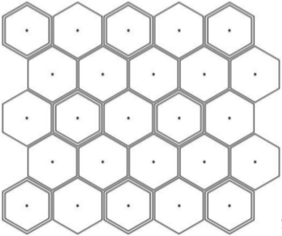
[5] E. Verlinde, JHEP 1104 (2011) 029, arXiv:1001.0785 [hep-th]

Walker position encoded on spin network

$\rightarrow$ From (4) and (13) we can compute the local entropy from a vertex as

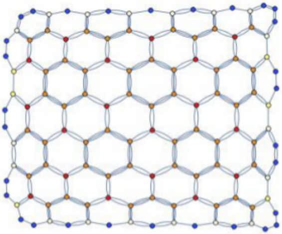
$$S_{E_n} = log\sigma - \frac{1}{\sigma}\sum_{m}^{N} j_{mn}(j_{mn}+1) log\left(j_{mn}(j_{mn}+1)\right), \quad (16)$$

where $\sigma = \sum_{m}^{N} j_{mn}(j_{mn}+1)$ of neighbour links. For example at a vertex $\{2,3,3\}$, $j = \{1, \frac{3}{2}, \frac{3}{2}\}$ gives $\sigma = \frac{19}{2}$ and $S_{E_n} = 1.06187$. At a vertex $\{2,2,2\}$, $j = \{1,1,1\}$ gives $\sigma = 6$, $S_{E_n} = 1.09861$ which is the maximal possible local entropy.



Entropy map

→ The local entropy at each vertex is color coded. From equation (15) a massless particle move on same color and a massive particle moves along constant absolute color differences.



Walker position topologically encoded

$\rightarrow$ The walker position, or the presence of a particle at one vertex is encoded by a triangle. Its move is a couple of 3-1 and 1-3 Pachner moves on neighbor positions, piloted by the walk probability

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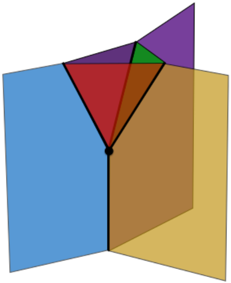

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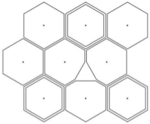

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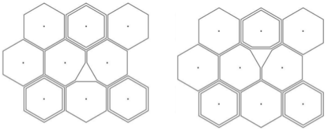

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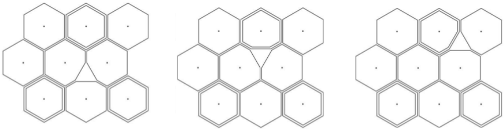

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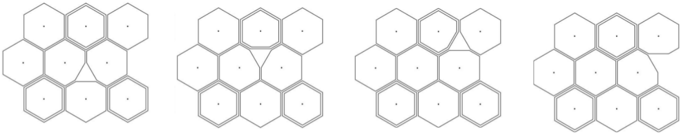

Black hole isolated quantum horizons

→ We can propose that DQW is the black hole quantum horizon, where the particle mass is the black hole mass in a random quantum walk on a fixed spin network.

→ Using results from isolated quantum horizons formulation of LQG applied to black holes [6]. And counting microstates associated to links of vertices of large valence. We see that

$$logN(A) = \frac{log(\phi)}{\pi\gamma}\frac{a}{4l_p^2}, \quad (17)$$

where $\phi = \frac{1+\sqrt{5}}{2}$ is the golden ratio.
→ Bekenstein-Hawking entropy is recovered by setting the Barbero-Immirzi parameter

$$\gamma = \frac{log(\phi)}{\pi} \quad (18)$$

[6] J. F. Barbero G. and A. Perez, arXiv:1501.02963 [gr-qc].

Discussion

→ Results from Loop Quantum Gravity suggest the interesting idea that we can apply the results and tools from quantum information and quantum computation to a quantum spacetime.

→ In this work we start a project to apply this tools like DQW to spacetime. We considered a DQW of a quantum particle on a quantum gravitational field and studyied applications of related Entanglement Entropy.

Discussion

→ With this we proposed a model for a walker position topologically encoded on spin network. This results in anomaly cancellation because the particle is no longer a point but has Planck scale size.

→ Relating this model with QuasiCrystal E8 model [7] are under investigation: the spin network can be choosen as the dual of a quasicrystal and the digital physics rules can be implemented by the quantum walk.

[7] F. Fang; K. Irwin, arXiv:1511.07786.

Discussion

$\rightarrow$ (arXiv:1602.07653)

Thank you.